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# Option value at risk and the value of the firm: Does it pay to hedge?

**by Roger J. Bowden***

*Decisions to modify the firm's natural exposure by using derivatives should be referenced back to the maximisation of corporate value. Firms typically have a natural exposure to adversity, costs that may start well in advance of a bankruptcy point. The implicit value of the resulting adversity or hazard options extends a long shadow over corporate value, even in better states, and this is what hedging is designed to neutralise. The framework is used to integrate corporate value maximisation, value at risk and expected utility theory. Value at risk can be regarded as a socially imposed device to neutralise the shareholders' limited liability exit option and will often result in over-hedging. It may not be optimal to hedge in adverse conditions: one should hedge the prospect but not the event. Modifiers such as leverage, exposure uncertainty, market incompleteness, competitive threats, and bank regulation can be explored within the same framework.*

**Key words**: Adversity options, capital adequacy, conditional value at risk, corporate value, non cooperative games, generalised value at risk, utility alignment, hazard options, hedging.

**JEL reference numbers**: D81: G10, G13, G30; M14.

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## Executive summary

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## I Introduction

Open interest and speculation apart, should the firm try to alter the natural cash flows associated with its core business and if so, under what circumstances, and how should it be done? Of course, option and other derivative dealers are expected to hedge as matter of neutralising their exposures to the instruments they write, but what about corporates in the non financial sector or in funds management? This paper seeks to reconcile several strands of development in the risk management literature that touch on such questions. One seeks to understand the reasons why firms choose to hedge some of their environmental exposures, with reference as to whether such a choice is ever in fact an optimal one, and the relationship to owner value in the theory of corporate finance. The second is the value at risk literature. The latter has tended to develop along separate lines, the first being concerned with value at risk as a diagnostic, and the second with using it as a welfare objective in portfolio optimisation and similar problems. And a third literature that also turns out to be involved is the much older one on utility theory and choices under risk. How can one make sure that risk preferences of managers are consistent with those of shareholders, i.e. ensure the correct alignment of managerial and shareholder utility functions for money?

The unifying principle suggested in the present paper is not so much value at risk, as option value at risk. The latter provides the market's valuation of the hazards in the left hand tail, i.e. the adversity zone. Option value at risk represents the market's valuation of the adverse payoffs inherent in value at risk or conditional value at risk. The value of the implied options provides the risk aversion element to the managerial utility function, so that an otherwise indeterminate notion of managerial utility becomes determinate. Once this is recognised, the appropriate managerial utility function reduces to a market value. This is just as it should be, for the ultimate management objective is simply to maximise the value of the firm, given the shareholder cost of capital. Thus if hedging is to improve the value of the firm it must be analysed primarily in terms of the value of the adverse options it removes or mitigates, balanced up against the shareholder exit option under limited liability.

Hedging rationales have usually been explored by listing arguments for and against. A consequence of the Modigliani-Miller (1958) capital structure propositions is that adding correctly priced derivatives to the firm's balance sheet (or off balance sheet) should add nothing to the value of the firm. The latter is the sum the market value of the natural cash flows and the market value of the derivatives, and that is that. A forward, for instance, has zero market value at inception. A variant of the same argument would say that shareholders should be left to arrange their own risk management portfolios and in many cases would prefer to do so. Indeed, to the extent that hedging is not core business, it can entail risks and costs to the firm from inadequate systems to manage it, misplaced reliance on external advice, or managers who think they can outguess efficient capital markets. On the other side of the ledger, hedging can preserve cash flow and liquidity for use as working capital, or to take advantage of adventitious investment opportunities. Hedging can also enable the firm to smooth progressive corporate tax rates, or to take advantage of proprietary information. For discussions on such non-neutralities see e.g. Smith and Stulz (1985), DeMarzo and Duffie (1991), Froot *et al* (1993), Nance *et al* (1993), Geczy *et al* (1996), Haushalter (2000). Empirical evidence suggests that hedging can succeed in raising corporate value (e.g. Allayannis and Weston 2001, Jin and Jorion 2006). And there remains the undeniable fact that hedging is very often required by debt covenants, either directly or indirectly and has been seen as lowering the firm's effective cost of capital just for this reason. The latter indicates, if nothing else, that there is a link with the value of the firm, so that the 'sum of the parts' hedging model referred to above is incomplete. There must be synergies, but if so what, and do they arise only in the levered firm?

Discussions of this kind are important in understanding the reasons why firms might elect to hedge as a policy, but they do not answer too many questions as to the how and when. What sort of operational criteria are appropriate: Should hedging be based on simple variance minimisation (as is often assumed); what if anything is the role of expectations; are options or forwards best, or what sort of payoff profiles should be sought for derivatives exposures? One suspects, too, that some of the alternative hedging explanations referred to above are simply different takes on the same general problem. What is needed is the kind of conceptual framework that leads naturally to the operational choices. The hedging objective criterion, for example, should be naturally dovetailed with the exigencies that motivate hedging in the first place, and as a consequence end up maximising corporate value.

Corporate value is a good starting point. Standard models of corporate value and of credit risk run in terms of implied option positions. For instance, equity holders have a real option inherent in limited liability, namely the ability to walk away from a failed firm (the 'exit option'). The option is effectively written by the debt holders and is reflected in the pricing of the firm's debt. The existence of this and other agency problems of the kind raised by Jensen and Meckling (1976), together with the option based models of Black and Scholes (1973), Merton (1974) and Black and Cox (1976), became a conceptual foundation for the modelling of credit spreads and credit ratings.

But are there hidden options in even an unlevered firm, and might these have something to do with the hedging argument? We shall argue that the answer is yes, and helps to account for the hedging phenomenon. The options themselves arise from the prospect of adverse outcomes that may threaten the firm's continues profitability or even existence, so we can call them hazard options. Via their staggered strike prices and market value, they cast a long shadow, affecting current management decisions even though the firm itself may not be on the brink of collapse or anything like it. Their payoff incidence may strike well be before the point of bankruptcy.

The hazard function in statistics gives the probability of an adverse event like death given that the individual has survived thus far - an analogy is with lifetables in actuarial science. A probability of this kind is in turn the basis for value at risk in financial management theory and practice. The VaR point is a value or cash flow point likely to be violated with a predefined probability. Conditional value at risk (CVaR) refers to the expected value of the loss, given that the VaR point is exceeded. However, Bowden (2006a) has pointed out that imposing administrative VaR and CVaR limits can lead to inconsistency, and a perverse implied utility function for money. A more embracing attitude is to suppose that there is a hazard area at which managerial or owner welfare takes a sharper dip downward, capturing the spirit of the generalised value at risk approach without many of its problems. But to do this is to recognise that the welfare function involved is virtually the same as the payoff from a hazard option. More precisely, the expected utility function is identical with the market value of the hazard option. In this approach, one can replace utility and value at risk with implied option payoffs and their market valuation. The risk premium of expected utility theory becomes embodied as the price of the resulting hazard option.

The corporate hazard that springs most naturally to mind is bankruptcy. It is a well recognised principle in the theory of capital structure that it is the prospect of bankruptcy costs, rather than the event itself, that constitutes a limit on debt. But the same contingency should affect the value of the firm as a whole, regardless of the way that it is financed. If an all-equity firm goes bankrupt, then the costs diminish the break up value. A structured approach is useful. It is as though the equity holders of the going firm have written options in favour of third party claimants such as accountants, lawyers, and statutory managements. In addition, there may be a loss of the real option to mothball and continue at some time in the future; this is a deadweight cost to going bankrupt. The value of these options detracts from equity value, even if the firm is not right now at the brink of bankruptcy – the shadow of such an event reaches forward and influences current corporate value.

However, hazards and their costs are by no means limited to a single point such as a point of final collapse. There is typically a range of critical points, each marking some new stage in corporate distress, starting with signals such as cash flow squeezes, profit warnings or credit downgrades, followed by customer migration. Events of this kind are generally signalled well in advance of an actual bankruptcy point. Such exposures can be regarded as option-style contingent payoffs in favour of competitors, corporate raiders, or other third parties. Indeed, even bankruptcy is not the ultimate misfortune, for increasingly there are limits to the Ltd. Directors may find themselves personally liable for company collapses through negligence or malfeasance, or in some jurisdictions, culpable incompetence. Thus it might be better to think not of a single critical point, but a whole range of them. In the case of a levered firm this might be observable in the form of progressive credit downgrades, but a similar sequence of underlying critical points or stages will exist for any firm, levered or unlevered. The resulting cascade of simple hazard options combine to generate a payoff that looks just like a conventional concave utility function with marginal utility progressively higher at more adverse outcomes. In some circumstances, one can call on the mean value theorem to replace the cascade of strike prices by a single equivalent strike price. This would be the true corporate value at risk critical point. Either way, the resulting corporate value shadow can start a long way ahead of prospective bankruptcy. Uncertainty about the precise costs of financial distress can make the shadow even longer, creating an additional motivation to hedge.

The identification with implied options provides something that conventional utility theory cannot, namely the value decomposition principle, under which one can assign a value to a component payoff by breaking it down into its constituents. The decomposition principle has become a cornerstone of structured financing, just as it was in corporate finance theory with the work of Modigliani and Miller and others on capital structure, or for that matter in options pricing theory. In a complete market it means that there is no longer any need to call on expected utility: one simply identifies the option equivalent utility function and proceeds to value any prospect by reference to its constituent option components. There is, in other words, a natural utility function to feed into programming problems aimed at determining optimal hedge ratios. Its asymmetry is in marked contrast to the variance or mean square error minimisation that one observes so often in practice, which throws out the baby of good times along with the bathwater of bad times.

The decision to hedge for an unlevered firm is revealed as a balancing act between: (a) Neutralising the value of the hazard put options; (b) the opportunity costs in doing so; and

(c) the value of the limited liability exit option, which is in fact a call option. Paradoxically, it may pay *not* to hedge near a crisis point like bankruptcy, but to commence hedging only further out. The reason is that if spot rates are low but expected to improve, locking in now at forward rates that are tied to current spot rates (rather than expected future ones), carries large opportunity costs. This will adversely affect corporate value to a greater degree than the value of the hazard option. Moreover, locking in at the current forward rate over-compensates, for limited liability means that after the bankruptcy point, shareholders are unconcerned about further drops. The protection offered by the forward against this contingency simply has no value to them. Use of forwards has the perverse effect of neutralising the value of the shareholder limited liability exit option.

If debt is added to the capital structure, things change for the worse so far as equity holders are concerned. When debt holders require hedging in the loan indentures, they shift the entire burden back on to the equity holders. The latter are now forced to pay what may be an uneconomic amount of opportunity cost in order to buy back (as it were) the hazard options. From the perspective of shareholders, such an outcome is a game-theoretic deadweight cost to using debt.

Much the same effect occurs in Basel and other capital adequacy regimes. Financial system regulators seek to endogenise the costs of the limited liability exit option by imposing capital provisions, or value at risk regimes, that considered in isolation might not be chosen by shareholders themselves. Value at risk itself stands revealed as a socially imposed device to cancel shareholder's right to walk away in the event of bankruptcy. From the perspective of shareholder value, value at risk might be seen as too tough a risk management criterion, forcing shareholders, via their managers, to hedge when they might not want to. But a more comprehensive theory of value would note that shareholder value can only exist in a social context. If it is necessary to hedge in order to ensure the consent of debt providers or financial regulators, so be it.

A number of other modifiers exist for the hedging decision. One of the most interesting is the game theoretic element that arises when a competitor also has the choice of whether or not to hedge, which may amount to a life or death decision, as our case study makes clear. It is shown that if a Nash equilibrium to the resulting non-cooperative game does exist, it is likely to favour both parties deciding not to hedge, even though the stand- alone prescriptions of corporate financed might call for hedging.

Choices as between forwards versus options for hedging can be illuminated by the same market value framework. Forwards have some distinct advantages over options. They require less information about the precise nature of the firm's exposure, merely that it is there. And they are a better way for handling serious risk aversion, where the private risk of the firm is greater than market risk as a whole. On the other hand, a motivation to use options is suggested by a uniformly diminishing marginal utility function on the part of managers for earnings or cash flow. Title to the higher cash flow can effectively be sold off by writing call options at higher strike prices, creating a bull spread. We suggest, however, that utility functions of this sort are in reality a misalignment between shareholder and managerial risk preferences – something that should be remedied by managerial reward structures if necessary. Similar remarks apply to managerial utility functions dominated by concern for their professional reputations as managers. They will act much like financial regulators, for they are unconcerned with the value of shareholder limited liability exit options. This is one possible explanation for the empirical finding that large firms tend to hedge more – they employ a cadre of professional managers.

The scheme of the rest of the paper is as follows. Section II establishes the basic model of hedging in terms of corporate value maximisation in the presence of both adversity options and the exit option. The central tradeoff off between gains and opportunity costs is laid out in terms of their impacts on the value of the firm. The natural welfare function for hedge ratio programming is laid out. Section III explores the precise relationship with value at risk and considers also the issue of managerial utility alignment. Section IV explores some modifiers, notably the existence of debt on the balance sheet, uncertainty as distinct from risk, incomplete markets, competitive strategies, and the choice between forwards and options. Section V comments on corporate risk culture, and makes some contact with prudential regimes such as Basel I and II. Section VI contains some concluding remarks.

## II Corporate value, hazards and hedging

*The natural exposure of the firm*

In this section it is supposed that the firm faces a single risk, the price or value of which will be denoted by $S$. For brevity, the symbol $S$ is also used as a name for the risk itself as well as the price or value. As earlier noted, the incidence point $S = P$ for the costs associated with financial distress can be significantly above the point $B$ of actual bankruptcy, even if $P = B$ remains a significant special case. For the purposes of the present section, the adversity point $P$ is assumed to be known, though the 'fuzzy' extensions are considered at later points.

Two options style exposures are of relevance, in addition to the operating cash flows from *S* over the coming period. It will be useful to think of them as distinct, with two different incidence points P and B, even if in practice there may be an entire cascade of such incidence points, extending down to and even beyond the point of actual bankruptcy. The first type of exposure arises in connection with incipient financial distress, with strike price *P*. One can think of this as a 'hard times' point. Adversity exposures start at this point, although their price shadow reaches forward to affect corporate value even where $S > P$. The second type of implied payoff is the ability under limited liability provisions of shareholders to walk away once bankruptcy has been declared, with strike price *B*. In what follows, we show how these exposures may or may not be mitigated by means of hedging. The adversity options create an incentive to hedge; the limited liability option an incentive not to hedge, so the hedge decision has to balance the one against the other.

In the exposition of this section, some simplifying assumptions are made:

(a) The firm is exposed to a single source of uncertainty *S*, e.g. a foreign exchange rate for an exporter or importer, or the commodity price for a producer or user, and the value of the firm is measured as increasing in *S*;

(b) A derivatives market exists in *S* independently of the existence of the firm;

(c) The probability distribution of the value or price *S* is independent of whether or not the firm chooses to hedge, i.e. is state independent with respect to the firm's risk management decisions;

(d) The adversity exposures have single point of incidence *P* and the costs of adversity are linear in *S* thereafter i.e. for $S < P$;

(e) The firm is all equity, there is no debt.

Subsequent sections relax these assumptions in various ways. In addition, the model that follows is a two period model, in which cash flow and corporate value effectively coincide, apart from centring and scale transforms. The complex issues that arise in multi-period models are raised at a later point.

The natural payoff profile of the firm can be written as proportional to

(1) $$(S-B)_+ + \beta(S-P)_- .$$

In expression (1), the notation $x_+$ *and* $x_-$ isolates the positive and negative values of *x*. In other words, $x_+ = xSF(x); x_- = xSF(-x)$ where the unit step function $SF(x) = 1 \Leftrightarrow x > 0; = 0,$ otherwise. The first component of expression (1) is the payoff exposure of shareholders in the absence of bankruptcy costs. The second is the adversity exposure, equivalent to a payoff from an option with strike price *P*. It can be assumed that adversity costs are a charge on initial capital or liquidity provided and the latter is large enough to absorb any such charges.

It could also be remarked that payoff (1) is dominated by simply holding a unit of *S* as a physical (ton of copper etc), so why go to the bother of holding a share in the firm, with its attendant adversity risk? This can be fixed by rewriting (1) more fully as

(1)' $\quad c + (1+m)(S-B)_+ + \beta^0 (S-P)_-$

where *m* is a profit margin, and the constant *c* represents the break-up value of the firm in the event of bankruptcy. The various parameters could be calibrated against each other by setting expression (1)' equal to zero at *S* = *B*. However, from the point of view of decision theory, expressions (1) and (1) ' are equivalent with $\beta = \beta^0 /(1+m)$, so we will proceed with the simpler form (1) with the understanding that the constant $\beta$ can incorporate a normal profit margin.

Using $x = x_+ + x_-$, expression (1) can be recast as

(2) $\quad (S-B) - (S-B)_- + \beta(S-P)_-$

The first term in expression (2) is linear in *S*. The middle term has the nature of a long put option profile. It partly compensates for the short put profile arising from the adversity option, which is the last term in (2). The overall value payoff profile from (1) or (2) is uniformly upwards sloping in *S*, with a lower convex kink at *S*=*B* and an upper concave kink as *S* =*P*. In what follows we value the profile.

The market value of the future payoff *S* is just $S_0$, the current value of the physical price. It is assumed in what follows that the carry cost of a unit of *S* is just the risk free rate *r*. In that case the forward parity applies:

$$F = S_0(1+r).$$

Minor changes will need to be made if the carry cost is not just the interest rate. For instance in foreign exchange contexts, where *S* is the price of a unit of foreign currency, the carry is *r* – *r**, with r the domestic rate and *r** the foreign interest rate.

The market prices the natural payoff (1) as long a call option with strike price *B* and short β put options at strike price *P*:

(3) $\quad V_u = \pi_c(S_0; B) - \beta\pi_p(S_0; P).$

The symbol $V_u$ denotes the value of the unhedged firm. In this market value expression, $\pi_c(S_0; B)$ denotes the price of a call option on *S* with strike price *B* when the current value of the exposure variable is $S_0$; similarly for $\pi_p(S_0;P)$ as the adversity put option. Other option parameters are suppressed at this point.

Alternatively, the market value in terms of payoff (2) can be written

(4) $$V_u = \frac{F-B}{1+r} + \pi_p(S_0;B) - \beta\pi_p(S_0;P)$$

Version (4) draws attention to the value of the limited liability exit option as $\pi_p(S_0;B)$. It also connects usefully with the current forward rate $F$ and enables direct comparison of the two put option prices. Absent adversity options, pricing the firm at the current forward rate (as the first RHS term) understates its true value. The reason is that the forward rate prices in the effect of events $S<B$ that are irrelevant for shareholders, because of their limited liability option to walk away (though see section IV for the levered firm). If we decide to hedge using the forward rate, we must also allow for the value $\pi_p(S_0;B)$ of the residual put facility, representing the value of the limited liability exit option.

The value of hedging can now be explored. Suppose initially that hedging is all or nothing: the manager hedges all the exposure by buying or selling forward at rate $F$, or else none. There are two cases to consider, corresponding to whether the current forward rate $F$ is greater than or less than the incipient distress point $P$. The latter case would indicate that the market as a whole is rather depressed, without as yet being a direct survival threat for the firm.

*Case (a) $F > P$*

Neither of the option exposures in (2) is triggered when $S$ is replaced by $F$, and the resulting current market value is given by

$$V_h = \frac{F-B}{1+r}.$$

The value of hedging relative to the unhedged position is obtained as

(5) $$V_h - V_u = \beta\pi_p(S_0;P) - \pi_p(S_0;B).$$

If $P=B$, hedging will be beneficial if and only if $\beta > 1$. The intuition is clear: if $\beta > 1$ then with the same strike price, the adversity options outweigh the value of the limited liability exit option. Conversely if the adversity exposures are not large, then the firm should not hedge – use of the forward rate effectively overestimates the true risk involved.

If $P>B$, then the strike prices of the two types of option differ. The adversity options will be further in the money and have a higher market value than the limited liability exit option. The incentive to hedge correspondingly rises, so that $\beta > 1$ is no longer a necessary condition to hedge.

In what follows, we assume that $P>B$, which amounts to saying that the costs of distress start impacting before the actual bankruptcy point.

*Case (b) $B < F < P$*

From expression (2), the hedged payoff is now $F - B + \beta(F - P)$ with market value

$V_h = \frac{1}{1+r}(F - B + \beta(F - P))$.

The value of hedging is

(6) $$V_h - V_n = \beta[\pi_p(S_0;P) + \frac{1}{1+r}(F - P)] - \pi_p(S_0;B).$$

Using forward parity together with the put-call parity relationship in options theory, the term in square brackets in (6) reduces to $\pi_c(S_0;P)$, the price of a call option with strike at P. Hence

(7) $$V_h - V_u = \beta\pi_c(S_0;P) - \pi_p(S_0;B).$$

The intuition is that relative to remaining unhedged, hedging at $F$ effectively gives the user $\beta$ call options on the beneficial zone $S_0 > P$. One could call these the hedging endowment.

If there are no adversity options involved, then hedging is not worthwhile. This can be verified by putting $\beta = 0$ in expressions (5,7). The intuition is that mentioned earlier, namely that fixing at $S = F$ protects against outcomes $S < B$ that it is not necessary to protect. Doing so neutralises the beneficial limited liability option inherent in $\pi_p(S_0;B)$. Using expression (7), one can show that a necessary condition[1] for hedging to be at all beneficial is that

(8) $$\beta > \frac{\pi_p(B;B)}{\pi_p(B;P)}.$$

It is not necessary to have $\beta>1$ to justify hedging.

On the other hand, hedging can be a bad idea in depressed times. Figure 1 illustrates, based on expression (7) and assuming $B<F<P$. It plots the incentive to hedge for different current state variables $S_0$, balancing up the values of the adversity and exit options. The

[1] Use an exact first order Taylor expansion of the function $\Delta(S_0) = \beta\pi_p(S_0;P) - \pi_p(S_0;B)$, from $S_0 = B$ as base. Observe that for any $S_0^* > B$, it must be that $\Delta'(S_0^*) < 0$, as $S_0^*$ is further out of the money for strike price B.

balance is affected by different values of β, reflecting progressively more severe costs of financial distress. It is the lower zone that is the 'no hedge' zone, and not the upper. The existence of such a 'no hedge zone' is another reflection of the effect earlier noted, that fixing at *F* protects against bankruptcy itself. That is not strictly needed: it is just the bankruptcy costs that need to be protected in such an event, and they are incorporated in the value of the adversity options. Figure 1 illustrates the adage 'hedge on the prospect, and not the event'. By the time $S_0$ gets too low, it is too late to hedge. The 'no hedge' zone shrinks if the adversity risk factor β is higher (β' in the figure).

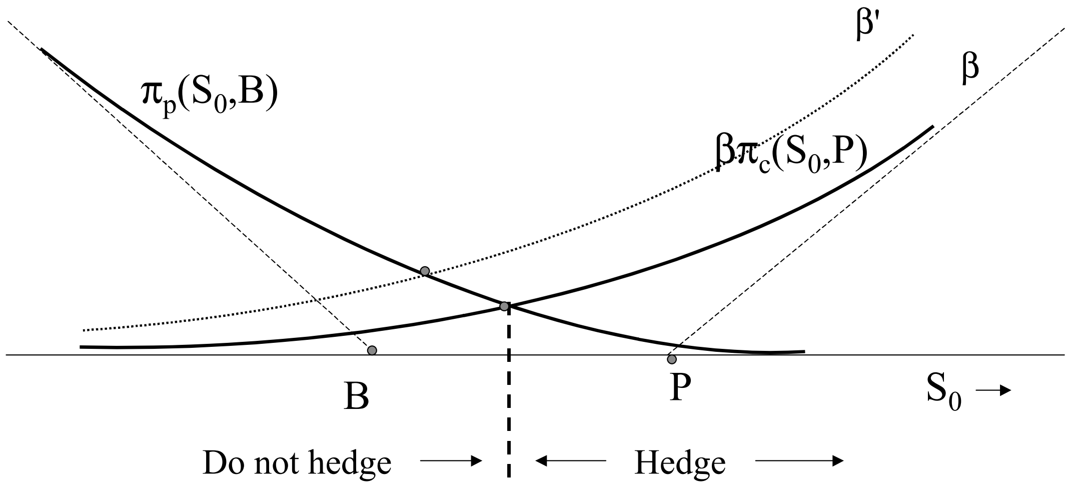


**Figure 1: Hedge zones assuming B<F<P**

Preceding development considered only the 'all or nothing' hedge. This is convenient to examine the value or otherwise of hedging, but it is not in itself a necessary restriction. By fixing only a proportion *h*, the user would achieve a conversion rate of

$$S^h = hF + (1-h)S$$

One would consider the normal case to be $0 \le h \le 1$, which might be imposed as some sort of administrative or prudential requirement, but cases such as $h>1$ also have some interest. The put or call options have now to be written in terms of $S^h$, which requires adjusting the volatility of *S* with the hedge factor *(1-h)*. In expressions (1) or (2) one replaces *S* with $S^h$ and values the corresponding options to obtain the value of the position for a given hedge ratio *h*. The object is then to choose the hedge ratio to give the maximum corporate value. If the hedge ratio *h* is allowed to exceed unity, the user is effectively shorting the physical *S* in favour of the forward *F*. The corresponding hedge problem turns out to involve call option values.

*Expectations and private information*

Expectations do not explicitly appear in the above expressions for the value of hedging, but they are there all the same. If the risk $S$ is priced in terms of a presumed capital market equilibrium, it will be true that

$$S_0 = \frac{\mu(S_0)}{1+r_c}$$

where $\mu(S_0) = E[S \mid S_0]$ and $r_c$ is a cost of capital[2] for investments in $S$. So

$$F = \frac{1+r}{1+r_c}\mu(S_0).$$

Using expression (6), the value of hedging where $B < F < P$ can be written

$$V_h - V_u = \frac{\mu(S_0) - P}{1+r_c} + \beta\pi_p(S_0;P) - \pi_p(S_0;B) - \frac{r_c - r}{1+r_c}\frac{P}{1+r} \qquad (9)$$

The last term in (9) can be treated as a constant for the purposes of a hedging decision. The first right hand side term indicates that hedging will be seen as more valuable when times are expected to improve relative to the distress point P. As it stands, however, expression (9) is saying no more than what hedging based on the market forward rate $F$ would reveal.

The significance of an expression like (9) is that it offers a window for the manager to consider whether his or her information is better than that of the market as a whole. Let $\mu_m$ denotes the manager's private estimate of next period's price $S$, and imagine that $\mu_m > \mu(S_0)$ so the manager is more bullish than the market. If the manager is correctly informed, then there is a bias towards remaining unhedged of $\frac{\mu_m - \mu(S_0)}{1+r_c}$. A hedging decision based on (9) with $\mu_m$ substituted for $\mu(S_0)$ will outperform a hedge decision based on the forward rate $F$.

*Multiperiod models*

In the multi-period context one has to distinguish cash flow from corporate value, separating them as two distinct variables. Starting at time $t = 0$, with initial capital $V_0 > 0$, there is stopping time $t = \tau$ at which time the firm becomes bankrupt (hopefully very long). The

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[2] In a complete market, for instance,

$$S_0 = \frac{F}{1+r} = \frac{E[\xi S]}{1+r} \sim \frac{E[S]}{1+r_c}$$

where $\xi$ is the market risk premium process such that $E_0[\xi] = 1$, corresponding to the Radon Nikodym derivative $dQ / dP$ of natural and risk neutral probability measures ( e.g. Duffie 1996, Appendix E) .

bankruptcy event is defined by $\{V_\tau = 0; V_t > 0, t < \tau\}$. At any time $t < \tau$, there are two sources of cash flow, bad as well as good. Adversity costs will occur if corporate value falls too low at any point of time. Thus a payoff of the form $(V_t - P)_-; P > B$ has to be added to the payoff arising from operating cash flows (measured here as a linear function of $S_t$ ). The current value of the firm is assessed as the expected discounted value of the sum of the two, taken up to the stopping time $t = \tau$. The discounting can be done with risk neutral measure at the risk free rate (complete markets), or as the natural expected value with cost of capital $r_c$ . The hedge ratio $h_t$ is now a function of time, regarded as an optimal control policy.

The multiperiod problem is mathematically complex, and analytic solutions are not possible. It seems reasonable to think that if the current value $V_0$ of the firm, together with $S_0$ , are jointly low enough to make bankruptcy a distinct possibility, then the firm might elect not to use forwards in the near months, especially if the adversity costs are not high (lower β range). As with the two period model, the use of forwards would forego the benefit of higher spot prices, and neutralise the benefit of the limited liability exit option. In such a depressed state, the optimal hedge strategy might have zero $h$-values in the near months, progressing though to unity in the more distant months.

An alternative approach is to maximise the change per unit time of corporate value, rather than its level. The incremental exposures over the coming period are a linear cash flow due to $S$ and payoffs associated with a contingent adversity event together with the possibility of bankruptcy. This is effectively a two period model of the kind outlined above. In terms of timing conventions, it has a correspondence with conventional value at risk procedures, considered further in section III.

Further hedge modifiers such as debt on the balance sheet or state dependence will be considered in section IV. However, the foregoing discussion of corporate value and implicit options suffices for the next section, which further considers the relationship to value at risk.

## III Value at risk, options equivalence and managerial alignment

The value at risk (VaR) portfolio diagnostic is a portfolio value or return critical value such that over a designated time interval, the probability of portfolio value outcomes below this level is a pre-set critical α, commonly 5 or 10%. A normative value at risk is a value $P$ such that portfolio value or return $S$ should not fall below this point with the given probability. In other words,

(10) $\quad F_S(P) \le \alpha$

where $F_S(\cdot)$ denotes the distribution function of *S*. In particular, conditional value at risk (CVaR) seeks to place a limit on the expected outcome given that it falls below the VaR point P:

(11) $E[S \mid S \leq P] > v$,

for some pre-assigned constant *v*. The idea is that a lot of sting can still remain in the tail, even if the VaR critical probability is within accepted limits. Many variants on these ideas have appeared in recent years; we can refer to them collectively as generalised value at risk (GVaR).

Value at risk and related ideas can always be cast in the form of expected values of the payoffs to put options. For instance, the distribution function used in VaR can be written as:

(12) $F_S(P) = E_S[SF(P-S)]$

where $SF(x)$ is the elementary step function such that $SF(x) = 1$ if *x >0, = 0 otherwise.*

Thus the given value at risk condition $F_S(P) \leq \alpha$ is equivalent to placing a bound on the expected value of the payoff to a binary put option that has value 1 whenever $S \leq P$ and zero otherwise. Similarly, conditional value at risk is the expected value of a payoff to a conventional put option. Condition (11) can be rewritten in such terms as:

(13) $E[S \mid S \leq P] = \frac{1}{\alpha} E_S[(P-S)SF(P-S)] \geq v$

where $F_S(P) = \alpha$ places the VaR restriction on the point *P*. The denominator on the right hand side of expression (2) is the expected value of the payoff to a put option with strike price at P.

Normative value at risk seeks to maximise the expected value of some payoff magnitude subject to contents such as (10) and/or (11). The most obvious maximand is just the expected value of *S* itself. This devolves the burden of risk aversion entirely to the GVaR constraints. Provided these are satisfied, the manager can otherwise proceed to choose investments that maximise the expected end of period value. Thus generalised value at risk can be thought of as providing a simple recipe for managerial decision making. Unless there is reason to think otherwise, just assume risk neutrality, except for outcomes below a comfort zone. Set the probability of such outcomes to some upper limit (value at risk) or put a ceiling on the expected loss if such outcomes should occur (conditional value at risk).

Details aside, this is not too bad a rule. Bowden (2006a) explored the effective payoff profile corresponding to VaR and CVaR portfolio choice programming problems. The former is equivalent to a lump sum penalty at the VaR point (*P*, say), so that the payoff profile is linear everywhere except for a vertical drop at *P* (see figure 2). Debt covenants may require penalties of this kind. Overall, the effect is that the payoff profile is not globally concave. In itself this is not troublesome. Conditional value at risk (CVaR) is more of a problem: the payoff profile can be discontinuous with a drop as *S* reaches from below the critical point *P*, which is not consistent with Von Neumann -Morgenstern utility theory. This can be fixed with a recasting of the conditional value at risk constraint to make sure that policies with a very low probability of outcomes being in the danger zone (<*P*) are not unduly penalised. When this is done, the effective payoff profile has the form depicted in figure 2, with a downward kink at the point *P*.

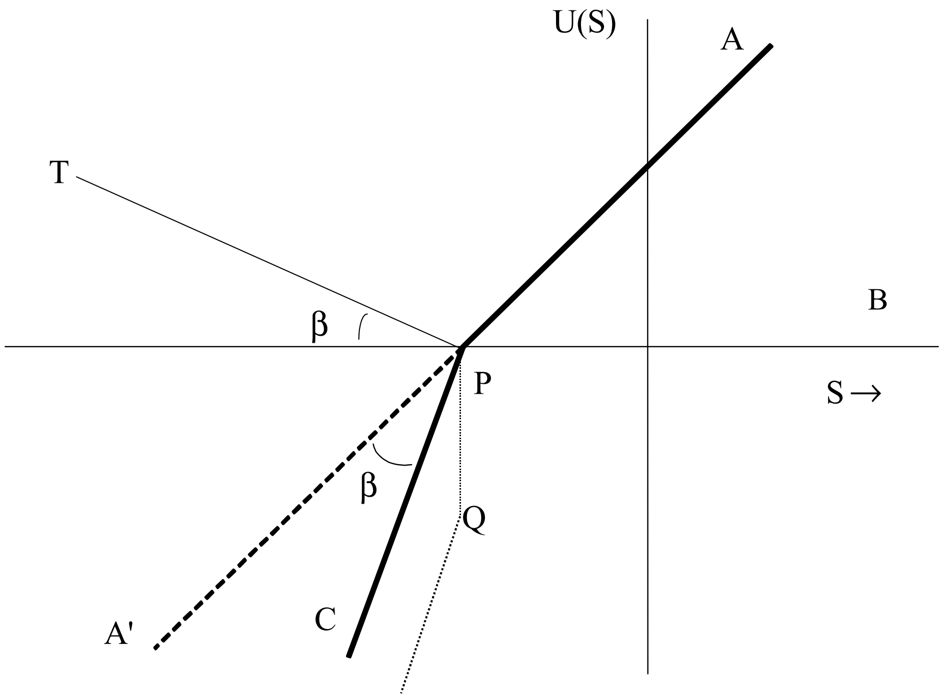


**Figure 2: Basic GVaR-type payoff profile**

In figure 2, the payoff is linear down to the VaR critical point *P*. It then kinks downwards with thereafter greater slope *(1+β)* representing a progressive penalty outcome. If there are administrative or other lump sum penalties for violating the VaR critical point, then the payoff profile takes a lump sum penalty PQ, as indicated. If there are no lump sum elements, the modified GVaR payoff profile is the locus APC. But this is the same as the original linear payoff less the payoff from writing β put options on *S*. The payoff to the holder of such options would be the locus BPT. Likewise, lump sum penalties could be thought of as being short in a binary option.

In other words, the GVaR payoff profile has been 'deconstructed' into a linear payoff together with a short position in options. Nor need one stop there. As earlier indicated, it might be more realistic to suppose that trouble comes more gradually. Starting at some initial signal point $P$, there might be additional adverse option triggers, at a series of strike prices $\{p \le P\}$. Added together, the payoffs from such a cascade of options can be made to generate any risk averse behaviour, in other words to mimic any given risk averse utility function in the adverse zone. Appendix A shows how any concave downwards payoff profile can be generated in such terms. From the pricing point of view, however, option cascades of this sort can be priced in terms of an equivalent simple option, so not much additional complexity is involved. The desired insight is that payoff profiles that look just like concave managerial utility functions can be valued in terms of option equivalents.

If value at risk can be portrayed in terms of put option profiles, then the relationship with the value of the firm becomes clear. One can identify the implied short position in put options with the adversity options of section II. The managerial utility function associated with value at risk should therefore be regarded as incorporating the value of the adversity options. The only significant difference is that generalised value at risk uses expected values of the payoffs rather than the market value of the equivalent options, as in section II of the present paper. For practical purposes we could consider expected managerial utility inherent in VaR as

(14) $$V_u = \frac{\mu(S_0) - P}{1 + r_c} - \beta\pi_p(S_0; P).$$

Here $\mu(S_o) = E[S \mid S_0]$, and $r_c$ represents a cost of capital for the firm, encompassing the systematic risk of the firm in normal operations. The put options may encompass lump sum payouts (VaR), conventional option profiles (CVaR), or yet other forms. The value $\pi_p(S_0; P)$ of the option component cannot be diversified away and is always unpleasant, no matter what the associated shareholder portfolio. It is a deadweight cost that detracts from the value of the firm. A device of this kind was employed in Bowden and Zhu (2006b) as a technique for objective smoothing in optimisation algorithms. The context in that case involved hedging cash flow in a study of foreign exchange hedging.

However, value at risk falls short of a complete theory of corporate value. Comparing expression (12) with (9), it can be seen that the GVaR utility functions are missing the value of the limited liability exit option $\pi_p(S_0; B)$, which acts to mitigate the downside exposure. In other words, the value at risk approach is a bit too tough. Managers driven by value at risk regimes will therefore tend to over-hedge. In doing so, they will be paying (with upside opportunity cost) for unneeded protection against the event $S < B$. Only to the extent that this event is associated with the adversity payouts will this be needed.

Two circumstances in which value at risk and corporate value might effectively coincide are as follows:

(a) Systemic risks and regulatory action. Financial industry regulators might superimpose an overriding GVaR objective in order to prevent systemic risks. In effect, they are saying that the limited liability exit option is unexercisable, a case of regulatory *force majeure*. By the same token, banks and insurance companies are also highly levered. This case will be considered further in section IV.

(b) Alternatively, actual financial distress costs are incurred only after the point of bankruptcy so that $P=B$, and moreover the option value of the costs outweighs the value of the limited liability exit option. Here one might also look at agency issues, with managers more averse to the event $S < B$ than are shareholders. In a depressed market (low $S_0$), shareholders are unlikely to lose too much value in bankruptcy – they will have lost that already as the share price fell along with $S$. Managers stand to lose a lot more – reputation, the costs of job search, or unemployment. In such case the net difference between the two $\pi_p(S_0;B) - \beta\pi_p(S_0;B)$ acts like a net adversity option.

*Managerial alignment problems*

As noted above, managers could become more risk averse than shareholders to more adverse states of the world. This is one possible way to rationalise the findings of Jin and Jorion (2006), who noted that large firms tend to hedge more than small ones. Large firms have more of a managerial culture, with a cadre of professional managers to whom reputation is important. If the firm falls into hard times ($S< P$), the managers would fear that this would be seen by subsequent employers as an adverse reflection on their professional capabilities. One could think of this as a justifiable disalignment of the respective utility functions of shareholders and the managers. So long as the professional managers succeed in adding their own value, shareholders would view with more equanimity the effective loss of the limited liability exit option.

More contentious is where the implied managerial utility functions exhibit diminishing marginal utility even in good states. Traditional utility functions concave throughout their entire length do not necessarily represent natural exposures, in the sense of the foregoing. Instead they may represent an overlay of managerial behaviour whose validity as to outcomes will have to be closely scrutinised by the firm's risk management committee for consistency with shareholder preferences. Diminishing marginal utility even in good times can result in a revealed preference for using options to protect exposures in bad times. Figures 3a and 3b relate such behaviour to option payoffs. In figure 3a managers assign no marginal value to outcomes $S > P_U$. The effect is though they have written a free call option with a strike price at $P_U$. In figure 3b, a cascade of such calls has been used to create continuously diminishing marginal utility (using the same methods as in Appendix A). Of course, the market as a whole does value outcomes above $P_U$, leading the more astute manager to simply sell off title to such outcomes, in the form of call options, generating corporate value as the resulting option premiums.

However, the lack of utility in the region greater than $P_U$ is not on all fours with the previous downside put options. In figures 3a and 3b the natural exposures for the firm continue to be the loci $AP_LBA$ and not $AP_LBC$. The utility function in the former case is naturally generated by the underlying natural exposure. The latter might represents an overlay of managerial preferences: so long as their jobs are not threatened on the downside, they are less preoccupied with the outcome on the upside.

If such behaviour exists it is a sign of possible misalignment between shareholders and managers. Empirical work generally suggests that people are risk averse and shareholders are mostly people. But it is a fallacy of composition to suggest that what is true for investor wealth, or the portfolio as a whole, must be true for each individual asset in that portfolio. The investor as company shareholder will want as much equity cash flow as possible, perhaps as a hedge against possible adverse cash flow from other assets. Risk enters via the cost of capital to be applied for this particular company, and once this adjustment is made, it is expected earnings that count, save in the adversity zone. In short, shareholder will want the firm to hedge only the unsystematic risk associated with financial distress. The systematic risk arising in more or less normal times is already allowed for in the cost of capital.

If the misalignment diagnosis is correct, bonuses or share option schemes can help to bring managerial and investor utility functions back into alignment. These can create personal opportunity cost for managers who might otherwise treat the upper zone as a free good. For related discussion see Campbell and Kracaw (1987), Bessembinder (1981).

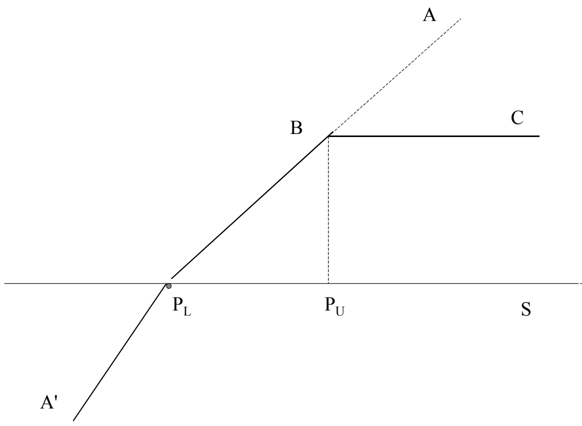


**Figure 3a: Call option equivalence**

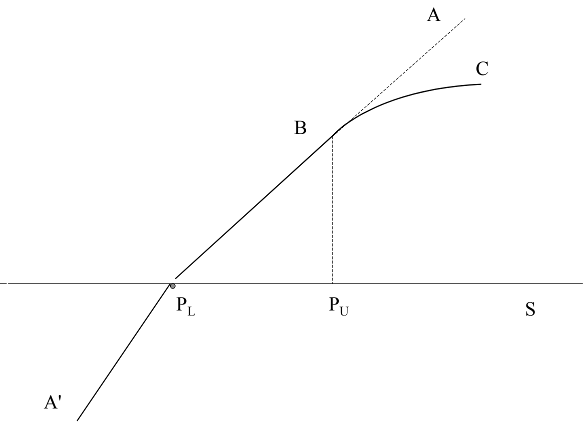


**Figure 3b: Call option cascade**

## IV Modifiers for the hedge decision

A number of factors will influence or modify the hedge decision. Some of these are hard to pin down; issues such as the 'risk culture' of the firm are considered in the next section. Others, such as tax smoothing, are standard in the literature and the present framework has little further to say about such things. In what follows we review some of the more structured of the potential modifying factors, in the light of preceding models. The special case *P=B* will be used as a simpler expository vehicle where a more structured approach is useful. Some of the modifiers have been well discussed in the literature, so the purpose is to briefly demonstrate how such effects relate to the framework of the present paper.

*The levered firm*

Hazard options are implicitly priced into a firm's credit rating. Debt covenants commonly required the firm to maintain liquidity and other designated financial management and capital ratios, with penalties of six figure sums or more for violations. If there is debt on the balance sheet, the creditors will directly or indirectly demand hedging, for two reasons:

(a) The debt holders bear the cost of the limited liability exit payoff enjoyed by shareholders, in the form of a break-up value that falls short of the nominal debt.

(b) The opportunity cost of using forwards should devolve to the equity holders. Debt holders do not get paid any more if *S* subsequently improves. So debt holders will ask the shareholders to pay for the hedging programme by foregoing income in good times.

In effect, debt covenants will require the equity holders to tacitly buy back the hazard options, using the foregone income should things improve in the spot market relative to the contracted forward rate. The debt holders do not share in the latter contingent gains.

Covenanted hedging of this kind will shrink any no-hedge zone. In the process it may amount to a welfare loss from the point of view of the firm as a whole – too much hedging. It amounts to a deadweight cost in the game between equity holders and debt holders. Further game theoretic elements are considered below.

In this connection, one should also note a potential credit problem with whoever is the counterparty for the forwards. If equity holders are forced into heavy use of forward because existing debt holders want to protect their own interests, there may be limits on their ability to do so.

*Incomplete hedges and basis risk*

In the foregoing, the existence of adversity options creates an incentive to hedge, regardless of expectations. The effect becomes weaker to the extent that the options themselves cannot be neutralised by means of market instruments. Suppose that the firm's exposure *S* now contains two components, in only one of which a derivatives market exists; or alternatively that there is a lot of basis risk in the chosen hedge instrument for a single risk. Imagine that one can decompose S as

$S = R + N$,

where a derivatives market exists in *R* but not in *N*. Without substantive loss of generality in what follows, we could assume $E[N] = 0$, the idea being that this is a random state of nature, adverse event or market development, or some other 'noise' factor.

Adding noise of this kind has several impacts on preceding development. These cover both the cost of capital used to value payoffs in normal times, and the valuation of the hazard options. If we assume that the noise *N* is a diversifiable risk to investors, the former will be unaffected, leaving only the hazard effect, which is not diversifiable.

Conditional on the value of *N*, a hazard option on *S* with strike price at $S = P$ is equivalent to an put option on *R* with strike price *P-N*, which would have market value $\pi_p^R(R_0; P-N)$, say. It is likely that the market would regard an option where *N* is unknown as having value at least equal to the natural expected value $E[\pi_p^R(R_0; P-N)]]$. Because the value of an option is convex in the strike price, it is not difficult to show that this exceeds the value of an option with $N = 0$, i.e. one based on *R* alone. Thus the first effect is that in the unhedged firm, the hazard options become inflated in value relative to the case where all risks are spanned.

On the other hand, the hazard risk cannot be reduced to zero by using the forward, as in preceding development, for there still remains an exposure to the noise element $N$. The fully hedged payoff is $S = F+N$, the hazard point is $S = P$, and the hazard payoff is triggered if $N < P\text{-}F$ . This formally equivalent to that on a put option on $N$ with strike price at $P\text{-}F$. The market could be expected to value it as at least equal to the expected value of such a payoff. Thus an increase in the noise volatility would mean that hedging was correspondingly less effective.

Counterbalancing effects therefore exist, and it is difficult to say a priori whether hedging reduces the hazard risk. A reasonable conjecture is that if exposures $R$ and $N$ are independent then fixing the value of $R$ by means of a forward is volatility-reducing on the total exposure $S$ and this will lead to hedging gains.

*Hedging as a competitive game*

A different source of uncertainty arises from competitor actions. This is an instance where the distribution of important state variables for the firm may depend on its own hedging decision and that of its competitor. In other words, the earlier state independence assumption for environmental uncertainty no longer holds

The issue can go two ways. Thinking as a manager, if my firm does not hedge and the opposition does, then it may be able to take advantage of my exposures to unfavourable states of the world. But on the other hand, if I hedge and my opponent does not, then the market may go the other way and the opportunity costs may encompass loss of market share. An extreme case in point was Bonlac Foods, a major Australian dairy exporter, which in 1996 hedged its exposure to the possibility of a yet higher Australian dollar, locking in an already high AUD several years out. Its competitor did not follow suit, preferring to remain unhedged. True to Murphy's law, the AUD then dropped precipitously, handing the competitor a huge strategic advantage, which it exploited ruthlessly, driving Bonlac into requiring a takeover bailout. Ironically, this was by Fonterra Ltd, the successor to the New Zealand Dairy Board, which earlier had done much the same thing but faced no domestic competition (Bowden 2005)

The lesson is that in the competitive environment, hedging with forwards is no longer a risk free outcome and the payoffs facing each player have to be adjusted as a consequence. Each player now has four payoff profiles, corresponding to the four possible combinations of strategies by each. These involve call options as well as the earlier put options. Thus if I hedge, but my opponent does not, then I have potentially written a call option in his favour, to be exercised if $S$ is higher than the current forward rate $F$. The resulting decision rule will incorporate this dependence, and the object is to seek equilibrium strategies to the resulting competitive game.

The equilibrium may well involve not hedging. To see why, note that if I do not hedge and my opponent does, then I am long in a call option which has more value at low $S_0$ for that is the region where I can do some damage to him if $S$ improves. The value of this call option may more than compensate for the value of the adversity option, leading me to choose the no hedge strategy. My opponent will think exactly the same way. Table 1 gives some illustrative value numbers: the first number of each pair is the payoff to player I and the second to player II. If both players did exactly the same thing, it would be to hedge. But if player I does decide to hedge, then the best strategy for player II is in fact not to hedge. There is only one Nash equilibrium and that is for both players not to hedge.

| I \ II | **Hedge** | **Don't hedge** |
|---|---|---|
| **Hedge** | 3,3 | 0,5 |
| **Don't hedge** | 5,0 | 2,2 |

**Table 1: Non cooperative hedging game**

Thus even if the optimal strategy in isolation is to hedge, companies that can damage each other may therefore seek out the no hedge strategy as the equilibrium, choosing the better of the worst possible outcomes. In addition, there is plenty of scope for managers to have different views. Bonlac's competitor, in terms of the earlier example, might have had access to better economic advice about the future of the Australian dollar. Knowing that a competitor may have some informational advantage is another motive for the manger to seek out more defensive strategies, which in the above instance involved not hedging.

*Hazard cost uncertainty*

The development of sections II, III assumed that the firm or the market can feasibly assign prices or values to payoffs that entail risk, whether this is market risk (foreign exchange rates, commodity prices etc), other economic risks, or internal risks to the firm. But when it comes to financial distress or bankruptcy, this may be easier said than done, for in most cases this is unknown territory so far as managers or shareholders are concerned. It may not be possible to assign definite probability statements to the costs that might arise – financial, reputational, stress-related, and so forth. This kind of uncertainty is perhaps related more to model risk than to market risk, for the firm's managers might find themselves unable to conceptualise the computation of contingent costs - they have no model to help them do it.

Figure 4 illustrates. It is based on expression (14) and assumes that a net adversity option exists with $P = B$. The incomplete model has been assumed and for ease of illustration, it is imagined that $\mu(S_0) = \mu$, constant. Reproducing expression (14), the value of the unhedged firm is given by

$$V_u = \frac{\mu - P}{1 + r_c} - \beta\pi_p(S_0; P),$$

where $\pi_p(S_0; P)$ is the value of the adversity options, while $P = 0$ in the figure. The bold curves (indexed by β) represent the natural value of the firm as a function of the current exposure price $S_0$. The value of the hedged firm is represented by the upward sloping line giving the present value at the forward rate. The firm will hedge if $S_0 > S_0^c$ and not hedge otherwise.

Model uncertainty is represented in figure 4 by varying the scale parameter β for the adverse option payoff profile. The firm is assumed to have imprecise knowledge of the probability distribution of β, so technically, this has more of the nature of uncertainty rather than risk. A range is possible, represented by $\beta_\ell$, $\beta_m$ and $\beta_h$ ( low, medium, high). With each we can associate a critical boundary separating the hedge – no hedge zones. Because the values of β can vary, the hedge-no hedge boundary becomes a fuzzy one, indicated with the shaded band A in figure 4. To be safe, the firm might well decide to select the higher, more adverse value of β. This will move the hedge boundary further to the left, increasing the motivation to hedge. The effect will be stronger when the expected value μ of *S* is lower, i.e. things look more pessimistic, for then the curvature of the adverse option value is greater at the boundary. At a higher value μ', less exposed to the adversity option, the zone of indeterminacy is smaller (shaded zone A' in the figure). The conclusion is that uncertainty as to what will happen in adverse states will bias the decision more towards hedging. This effect will be more important if the outlook is pessimistic to begin with.

Note finally that decisions as to the use of forwards may not require any very precise knowledge of the costs of adversity, only that they are likely to lie within a sensitive range.

They have the advantage in this respect over options as a risk management tool. Neutralising the adversity options by means of the latter would require fairly precise knowledge of P and β. In addition, the firm using forwards can benefit from the likelihood that the market as whole may be less risk averse, even risk neutral. If this is true, then using the forward enables the firm to effectively shift its risk to the market, diluting it in doing so in the greater size of market positions as a whole.

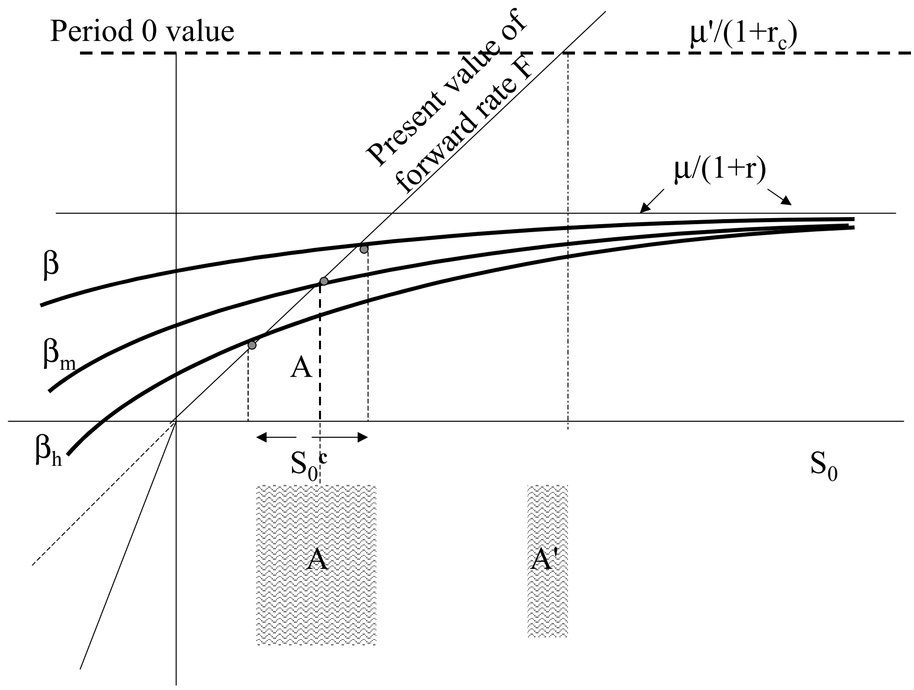


**Figure 4 Zones of uncertainty and the bias towards hedging**

*Hedging and liquidity*

Assured prices also mean assured cash flow, so using forwards means that corporate decisions and current liabilities management are that little bit easier. Corporate investment planners can sleep easier knowing in advance that the cash flows will validate their projections and justify the investment project. Likewise, selling call options can generate cash that may be needed to avoid funding short term working capital needs by means of expensive borrowing, especially if current spot prices are low.

A counterargument to fixing is the opportunity cost of not being able to take advantage of higher spot prices in the future. Moreover, the arguments from liquidity and certainty can be overdone. Some firms have ready access to liquidity from a parent. Thus Metallgesellschaft MG, which got into hedging difficulties in the early 'nineties, was owned by a consortium of financial institutions. In addition, hedging can itself create cash flow difficulties if it is done via futures, which required constant margin calls. Metallgesellschaft ended up needing the support of its parents for this reason.

## V Risk culture and prudentials

In many cases, managers will have only a vague idea of the extent or shape of the adversity payout. The latter is a latent exposure; everybody knows it is there, even if measuring it is problematic. But it need not particularly matter that one is unable to price very exactly the hazard options. Just knowing they are there may be half the battle, because the exposure can be used to establish common ground for discussions on corporate attitudes to risk. The frequently encountered notion of a 'corporate risk culture' can be justified in such terms. Firm A might know that it has more of a natural exposure to adverse events or potential distress costs than does firm B in the same industry. Its adversity options will have a higher market value, manifested in a lower share price if the firm remains unhedged. This being the case, firm A will have more of a culture of risk aversion than does firm B. The idea of a corporate risk culture might at first glance seem vague, or a way of sweeping things into the too hard basket, but it can make sense in terms of maximising the value of the firm by minimising the value of the adversity options.

Turning to prudential issues, Basel capital adequacy conventions for financial institutions run in terms of the ratio of capital to risk weighted assets. The risk weighting of assets could be viewed as an attempt to net off the value of the adversity option associated with institutional default, in effect charging more capital for assets that have higher probability of precipitating adverse outcomes. Probably the closest officially sanctioned risk management policies to the adversity option approach are those that encompass value at risk and its extensions. As mentioned in section III, value at risk itself runs in terms of probabilities, but conditional value at risk does entail expected values corresponding to option payoffs in adverse states, even if they are not market values.

If the value of the adversity options could be assessed with a reasonable degree of precision, it would provide an appropriate decision criterion for the capital adequacy debate. A financial institution would set the capital adequacy bound in terms of the adversity option value to capital ratio. Investment policies would have to ensure that

$$\frac{\pi_p(S_0;P)}{K} \leq \gamma \qquad (13)$$

where $K$ is the amount of tier 1 capital, the current state $S_0$ represents a vector of market prices and other current state variables, and $\gamma$ is the designated capital adequacy bound.

Note that the bound (13) ignores the value of the shareholder limited liability exit option. Bank regulators might well take the view that shareholders should not be allowed to factor in the mitigating effect of the limited liability option. If so, this would amount shareholder being asked to pay for the public costs of systemic risks. The matter is analogous to debt holders protecting their own interests by requiring the firm to disregard the benefit of the limited liability option in determining hedging policy.

## VI Concluding remarks

The existence of latent adversity or hazard options provides a framework for thinking about risk management, encompassing value at risk and managerial expected utility within an overall framework of corporate value maximisation. The particular issue considered in the present paper concerns the decision to alter natural exposures by means of hedge instruments. The ultimate recommendation is to do so if it enhances the market value of the firm. This is a matter of balancing up the gains from neutralising the latent adversity options as against the opportunity cost of hedging, on the one hand, and the limited liability exit option in the event of bankruptcy, on the other.

Very often hedging will indeed be justified, especially if the firm is content to take the view of the markets as to the future course of spot prices. But it can make perfect sense not to hedge, even if things look bad, where there are negative carry costs in forward pricing, or where the firm thinks that the spot prices will improve. The latter may not necessarily be a matter of acting on a wing and a prayer – the firm may know more about the prospects for spot prices than does the market as a whole.

Value at risk may in some contexts lead to over-hedging, relative to what corporate value maximisation would require. If shareholder enjoy an unfettered option to walk away, use of forwards can be an expensive way of protecting against adverse outcomes, for it protects against states of the world in which shareholder would other exercise the exit option. Value at risk can instead be viewed as a defensive response from debt holders or systemic regulators, concerned to see that shareholder will not be able to exercise the exit option quite so freely.

The hedge decision is modified by a number of factors. As noted, the levered firm will alway hedge more, because it is forced to do so by debt covenants. Uncertainty as to adversity exposures will also bias a hedge decision. On the other hand, competitive pressures may lead to a realisation that the hedge strategy is not optimal in certain states of the world, and the firm will remain unhedged.

The most important thing in all cases is to have a reasonably coherent conceptual framework to guide the hedging decision and corporate risk management policy as whole. It would be of interest to lay out more extensively the relationship between Basel type capital adequacy and other prudential regimes to the adversity option framework. The one might be used to better inform the other. A growing problem with risk management as a whole is the existence of so many rules, regimes and recommendations. The risk management industry is looking increasingly like the Tower of Babel, with a consultant camped on every stair. To be sure there are other agendas, notably systemic stability. But one would like to think that if risk management decisions were guided first and foremost by corporate value maximisation, this would help to assess the value added in terms of other agendas.

## Appendix A: Option cascades and utility functions
(Section III)

Suppose that financial distress costs are is triggered at a series of points $p \leq P$. Denote the payoff to each short put option component as

(A1) $U_p(S) = -\max(p - S, 0)$

where $p$ is the strike price. The payoff to a continuous option cascade at a range of strike prices $L \leq p \leq P$ can be written

(A2) $V_P(S) = \int_L^P \theta(p) U_p(S) dp$,

where $\theta(p)$ is a semi-positive weight distribution, and the lower limit $L$ may be (-∞). Also write the cumulative weight function as $\Theta(S) = \int_L^S \theta(p) dp$.

*Remark*: An expression of the form (A2) could be given an alternative interpretation in terms of either probabilities or fuzzy membership functions, in the sense of Zadeh (1965, Zadeh and Bellman (1970), and Zmeskal (2001).Suppose that there is in fact just one critical point $S_c$ dividing the $S$ axis into the good zone ( $S>S_c$) and the bad zone ($S<S_c$). Managers and shareholders are unable to assess in advance exactly where the critical point $S_c$ will be. However, given any potential outcome $S = p$, they can assign a good zone membership value, or equivalently the probability that the critical point $S_c$ is less than or equal to the given value p. Then $\theta(p) = \Theta'(p)$ can be interpreted as the probability that

$S_c = p$. Expression (A2) becomes the weighted average of option payoffs with strike prices at possible alternative values of the critical point S.

Continuing, it follows from (A1) and (A2) that

(A3)
$$V_P(S) = \int_S^P \theta(p)(S-p)dp : S > L$$
$$= \int_L^P \theta(p)(S-p)dp \quad S \le L;$$

The first two derivatives of $V_P(S)$ are given by

$$V_P{}'(S) = \Theta(P) - \Theta(S) \quad \textit{for } S > L$$
$$= \Theta(P), \textit{constant,} \quad \textit{for } S \le L,$$

$$V_P{}''(S) = -\theta(S) \ \textit{for } S > L$$
$$= 0 \ \textit{for } S \le L.$$

The complete payoff profile would be

(A4) $\quad U(S) = S - P + \beta V_P(S)$,

where the compound option $V_P(S)$ replaces the simple payoff that appears in figure 1. In what follows we shall refer to β as the scale parameter, and to $\theta(p)$ as the shape function or parameter. The complete payoff profile $U(S)$ is upward sloping and concave, though not strictly so.

*Example 1*

$L = -\infty$ and $\theta(p) = \lambda e^{-\lambda(P-p)}$, an exponential density.

Then for S<P,

$$V_P(S) = S - P + \frac{1}{\lambda}(1 - e^{\lambda(S-P)}).$$

This is illustrated in figure A1 (with a negative sign to denote the third party payoff). Also depicted is the complete payoff profile $U(S)$.

*Example 2*

If $L = -\infty$ *and* $\theta(p) = 2c$, constant, then

$$V_P(S) = -c(S-P)^2; S<P.$$

This can be generalised to cover any power exponent $\kappa$. The case $\kappa = 2$ and $P=E[S]$ would correspond to using the semivariance as a downside risk metric. This can cover situations such as pointed out by Yamai and Yoshiba (2002), where the expected shortfall implicit in standard CVaR fails to eliminate the tail risk beyond a specific threshold (i.e. very long tail effects); see also Ferreira and Goncalves (2004). In term of option equivalence, the effect would be as though the manager had written power options on the downside. With $L = -\infty$, as above, the negative slope is continually increasing as S decreases. If $L > -\infty$, the slope becomes constant after $S = L$.

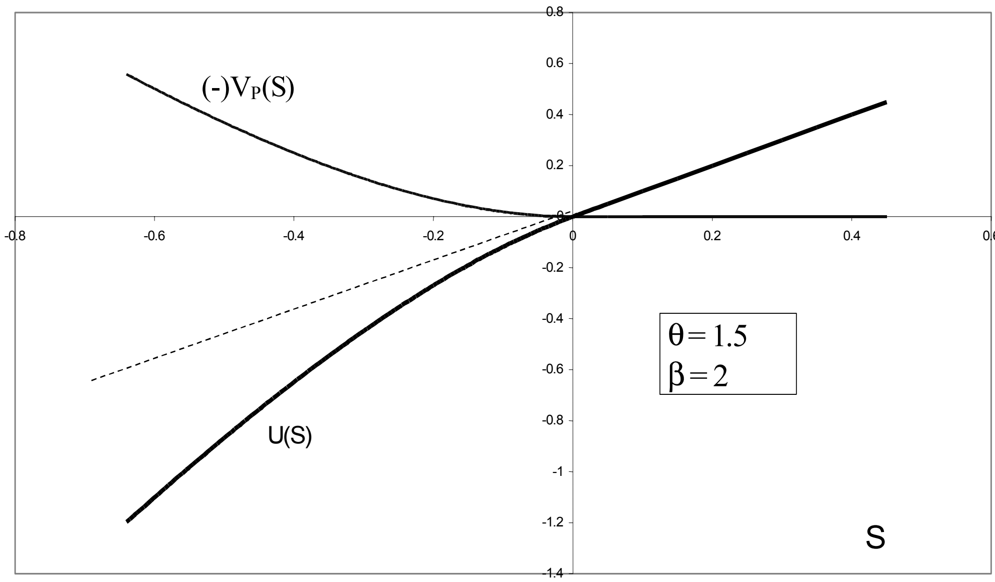


**Figure A1: Exponentially weighted option cascade**

The use of option cascades such as (A2) to mimic conventional utility functions is a potentially useful programming device to generate smooth objective functions. From the pricing point of view, however, the net effect is not too much different from a simple option. To see this, let $\tilde{\pi}(S_0; p)$ denote the premium of the simple put option at strike price $p$ when the current value is $S$. Then the valuation of the compound option payoff $V_P(S)$ is given by

$$\pi(S_0; P) = \int_L^P \theta(p)\tilde{\pi}(S_0; p)dp$$

Suppose in particular that $\int_L^P \theta(p)dp = 1$. Using the second mean value theorem of differential calculus,

$$\pi(S_0; P) = \tilde{\pi}(S_0; p^*)$$

for some $p^*$ between $L$ and $P$ that in general will depend upon $S_0$ as well. In other words the compound option is priced as though it was a simple option with effective strike price at $p^* < P$. In the text we exposit things in terms of the simple option but its strike price $P$ can be thought of as the centre of gravity for a complete range of discomfort points and associated payoff profile (i.e. reset $P = p^*$, in effect).

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