Derivatives, debt and defined benefits in national retirement policy

Roger J. Bowden[1]

[NZ context]

- *As the cost of funding baby-boomer retirement under defined benefit schemes has become apparent, the resulting paradigm shift to defined contribution - but undefined rewards - has left pensioners exposed to performance, credit, and time to death risk, looming ever larger as life tables lengthen. It is argued that defined benefit schemes can be designed off the back of high grade debt issuance programmes, that might be used to finance long term public asset vehicles and resolve agency problems in public retirement provision. Such schemes provide the long-term certainty that assists individual lifetime planning via the consumption floor, and ultimately reduces the residual provision burden on the government.*

- *Financial technology can help. Derivatives can be designed to enhance otherwise unexciting coupons and to correctly align preferences as between income while still alive and bequests. Lifetime reinvested coupon options and annuity swaps utilise market pricing to provide unambiguous pricing benchmarks and a necessary underpinning of lifecycle planning certainty. The result is a flexible mix of private and public provision of old age income assurance that exploits the externalities of a well designed system of public debt. If NZ real interest rates are high, why not use them constructively?*

- *The underlying debt instrument could be issued by a public long-term asset vehicle (LTAV). Some possibilities are:*
  - *Sequestering elements of the Government's capital programme in the form of a dedicated Public Infrastructure Fund. The proposed fund would concentrate its resources on infrastructure with measurable economic rewards. It could be used as a partnership vehicle with private enterprise e.g. in the construction & operation of toll facilities.*
  - *The use of debt to lever up the National Superannuation scheme. This would provide a natural performance benchmark, as well as further equity distributions from riding the long-term risk premium.*
  - *SOE debt, consolidated if necessary.*
  - *Student loan debt appropriately sequestered and packaged.*

- *Existing attitudes towards NZ public debt are unduly cautious, for understandable historical and defensive reasons. Such perceptional problems could be resolved by the use of LTAV's. The result would be a more a more effective and transparent public finance system, in which implicit public liabilities become explicit. Moreover, NZ public debt is currently short maturity by international standards. The issue of longer maturity public debt would facilitate a more efficient financial system as whole.*

- *Although the primary physical debt instrument would be public, a role exists for the private sector in enhancing the debt via the proposed derivatives. An agency role could also exist where government guarantees apply to private sector debt.*

- *Application points exist for tax incentives to assist with equity and efficiency. However this is not a necessary feature.*

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Lifecycle derivatives, defined benefits and retirement income assurance

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Abstract

As the cost of funding babyboomer retirement under defined benefit schemes has become apparent, the resulting paradigm shift to defined contribution - but undefined rewards - has left pensioners exposed to performance, credit, and time to death risk, looming ever larger as life tables lengthen. It is argued that defined benefit schemes can be designed off the back of high grade debt issuance programmes, that might be used to finance long term public asset vehicles and resolve agency problems in public retirement provision. Derivatives can be used to enhance coupons and to correctly align risk preferences as between income while still alive and bequests. Variable lifetime reinvested coupon options and annuity swaps utilise market pricing to provide unambiguous pricing benchmarks and a necessary underpinning of lifecycle planning certainty. The result is a flexible mix of private and public provision of old age income assurance, that exploits the externalities of a well-designed system of public debt.

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I Introduction

As the babyboomers near retirement, assurance of a decent standard of living has become one of the most debated economic issues of recent times. There is a measure of agreement that while a role for government is appropriate, the accumulation of unfunded public liabilities may place too much of a burden on the next generation and all round, is not a good idea. Yet there is much less agreement as to what the optimal scheme should look like, even absent the problem of transition from existing arrangements. Some countries and states have put into place funded schemes via a public investment fund that essentially mimic private investment funds, while others have adopted taxation incentives to stimulate contribution to private sector investment funds. Such arrangements differ from the old schemes in several ways. Typically, they do away with the idea of defined benefits, i.e. promises to pay an assured amount in the future. They may replace this with defined contribution schemes, exposing the retiree to the subsequent performance of the fund and its investments, with or without a public bail out provision in worst case scenarios like collapse. Thus in recent years there has been substantial swing toward 'unit linked' or 'equity linked' arrangements, in which what can be taken out is simply what has been earned on what has been put in - in the case of lifetime annuities, less heavy administrative, fund management and insurance costs (SEC 2004). Alternatively, as in the public funds, there may be little assurance that even performance provisions will apply. The amount to be distributed is not fixed by any pre- set formula, with an ill defined moral commitment on the part of the government to continue to make provisions, and exposure to the policy inconsistency problem.

Further risks are provided by the preference on the part of governments that all investment rewards should be distributed in the form of annuities rather than made available as a lump sum on retirement; indeed this is legislated in the case of the current UK scheme. This effectively gives an official mandate to the duration mismatch of private arrangements, for contributions are made up front and benefits are only received many years later. Hence there is not only heavy performance risk arising from the market risk of investments, but also credit risk, hardly reassuring in the light of recent financial difficulties of insurance companies and similar providers. Venerable does not mean permanent, the most prominent recent instance being the forced closure in 2000 of Equitable Life Assurance, after 288 years of business in the UK. For a general review of annuity schemes in this context, see Brown et al (2001).

The present paper adopts the starting point that annuities (suitably enhanced) are indeed an appropriate mechanism for the distribution phase and that the responsibility should be at least partly placed on people to provide for their own retirement. We depart from prevailing wisdom by suggesting that an important role remains for defined benefit schemes. Planning for retirement requires a measure of certainty as to investment outcomes or government assurances, even if the tranche so secured is only part of the individuals' portfolio of retirement assets or public promises, and used to assure the 'consumption floor'. Unit linked schemes do not provide the needed level of certainty. Mitchell *et al* (2001) have drawn attention to the substantial risk premium that people would be prepared to pay for an assured income in the face of an unknown time to death following retirement (these days, increasingly long and variable). They also draw attention to a disconcerting variation in the expected net present value of historical annuity payouts by insurance companies, and also to the apparent deadweight costs of this form of contract. One implication is that if people had a basic tranche of retirement income that was defined benefit in nature, then they might adopt a more effective investment policy for the remainder.

'Defined benefit' has had a bad press - it need not imply open ended public or private liability. Investing in a very high grade fixed coupon government or corporate bond will always provide an assured defined benefit. It is also easy to price. There are no difficulties with ascertaining the appropriate cost of capital for an estimated net present value calculation; the latter can be done off standard market instruments rather than idiosyncratic institutional portfolios. Provided the bond has not been issued solely for the purpose of providing retirement income (i.e. the bond has a solid asset backing or achieves some other economic purpose), there is no deadweight economic loss in such an arrangement.

The problem is that such bonds are unexciting in their investment returns; it is in seeking higher coupons on private sector instruments or investments that older people get into so much trouble. Moreover, the temporal pattern of payments does not correctly align with the requirements of retirement and death. The only way to assure oneself of income support right to the end means that the bequest motive is automatically treated as of equal importance with the retirement income motive. A menu of choice would be more useful, enabling the individual to shift benefits and burdens forward or backward according to intertemporal risk preferences.

The intertemporal risk shifting and coupon enhancement can be accomplished using financial technology. The volatility inherent in long duration bonds, as well as swap driven rearrangements, can be used constructively to align preferences as between cash flow support so long as still living, and the bequest motive when the end finally does come. Achieving this does not at all violate the 'no free lunch' principle of efficient financial markets. Financial instruments can be designed to rearrange temporal profiles for both income and risk. Using annuity swaps is a rearrangement of certain cash flows. And by giving up a measure of certainty in bequest, a greater certain income can be attained while still alive by reinvesting option income to create higher coupons. In this way the user can trade risk for income certainty.

The approach taken highlights the multifunctional role of high grade long term debt. Raising government debt in different maturity bands is not solely a matter of perceived financing costs at the time; there are some important externalities involved. The use of treasury interest rates as market reference rates is well known, and the same function could potentially apply as a benchmark for the payouts achieved in annuity schemes. Other uses are familiar in markets for risk, e.g. the use of long bonds as the physical in futures contracts, or the engineering of long dated high grade zero coupon bonds and their use in capital-guaranteed equity investment schemes. We point up another in connection with the underpinning for realistic defined benefits retirement incomes, namely their use as the physical in income enhancing derivatives. Externality benefits of such kinds should be factored into public sector debt management decisions, for they diminish the effective costs to the government of retirement provision. Decisions as to how much government debt to issue, or in what maturity bands, have to consider more than just the interest cost, especially if the formal costs of funds difference amounts to no more than a few basis points. Public debt programs are not zero sum games. Nor need the debt be nominally that of the government – the constructive use of a government guarantee in appropriate circumstances could help to underpin the credit status of debt nominally issued by the private sector, such as student loans.

The scheme of the paper is as follows. Section II establishes some background. A short discussion of portfolio and cash flow requirements precedes a review of difficulties with existing retirement arrangements, notably the adverse exposures to credit and performance risks, and the lack of common ground for pricing. What is needed in this respect is a high grade debt instrument to act as benchmark and to serve as the physical for derivatives based enhancement programs. Section III establishes generic types of derivatives that between them can cater for income enhancement and a menu of choice as between income and bequest motives. The first type is lifetime annuity swaps, essentially the equivalent of insurance industry annuities but with a more unambiguous pricing benchmark based on swaps pricing. In the second category are the variable lifetime coupon options (LCO's), wherein the coupons are enhanced by successive reinvestment of the premium income and the strike price is used to balance income and bequest motives. Payoffs are compared. Technically, there is no advantage of one over the other, as both as derived from efficient market pricing. But together they provide a menu of choice, especially where intertemporal income and risk preferences do not coincide with those of the representative market agent. The LCO's turn out to be more robust to the adverse selection problem than do the swap based enhancements. The section concludes with a brief discussion of institutional provision and credit aspects.

Section IV turns to the physical debt underpinning and the extent to which the requirements as to credit and maturity are provided by existing private and public originated debt. While a role might exist for general purpose private and government debt, it is suggested that public long term asset vehicles (LTAV's) would provide desirable asset cover, matched as to duration, as well as helping to solve policy consistency problems. LTAV's could encompass infrastructure funds, dedicated retirement investment funds and other government agencies with a natural backing of long term asset cover, or could extend to the packaging of semi-government or private debt with attached government guarantees. The use of dedicated LTAV's would help to resolve agency problems of retirement provision from general government revenue, where the liability is not registered as such, and is subject to agenda conflict down the track.

Section V contains some concluding remarks concerning the role in lifetime portfolio planning, further aspects of public finance and the role of universal retirement planning in this context. It is suggested that the use of LTAV's and their debt programmes to underpin enhancement-based retirement schemes leads to a mix of public and private provision that is more transparent in its pricing, equity and moral obligations than are existing arrangements.

II Motivation: lifecycle planning and institutional provision

2.1 Personal portfolio and income risks

Requirements and risk exposures of retirement support can usefully be set within an overall process of lifetime consumption and portfolio planning. It is convenient to think of this in terms of three phases of lifecycle portfolio planning.

(a) An initial accumulation phase in which asset growth is partly a consequence of saving from salary and wage income. Consumption is at least partly supported by returns from human wealth. Investment mistakes that might otherwise depress present and future consumption can likewise be buffered by the prospect of future returns from human wealth.
(b) A shorter transitional period of consolidation and review, near or following retirement from active labour force participation or other income generating activities, in which future needs are reassessed in the light of wealth and health at the time. At this time

decisions may be made that lock into one or other investment modes. The general issue of investment performance risk is an important consideration. It applies to the portfolio as a whole, as well as to the choice of individual investment or asset classes. Part or all of the portfolio may be converted to income for present and future consumption needs, so that this is conversion of prior capital gains to ongoing income.

(c) The final stage is where the sole support is from financial wealth and this can be subject to rundown to support old age, either by intention or by force of circumstances. This phase is not foreseeable as to length. In this phase, the ‘time to death’ risk becomes a significant consideration. Time to death risk has assumed greater importance in recent years, for people now live longer and the conditional survival densities are more dispersed (i.e. they die further apart in time).

The needs of the elderly in phase (c) can be formalised in a somewhat parallel fashion to the risk management of financial institutions. In the latter case, value at risk (*VaR*) sets boundaries on allowable institutional net worth outcomes in adverse states of the world; investment and trading activities are designed to minimise the probability of the values at risk. One could certainly apply a model of this kind to investment activities of the individual, though we later suggest it might be a more useful way to approach the bequest motive. However, the consumption support needs of personal old age can be better thought of in terms of cash flows at risk (*CaR*), which has acquired some currency in corporate circles as well, and indeed the ideas are similar. There is a minimum cash flow required to support a basic life style, the ‘consumption floor’ (Brown *et al* 2001). The latter might well depend upon financial history as well as personal preferences and circumstances. Nevertheless it is seen as important to at least achieve this in old age. Beyond that, a variety of behavioural preferences can apply (e.g. Barsky et al 1997). The implied utility function is similar to those employed in *VaR* studies (Bowden, 2004), in that utility drops sharply past a certain subsidence point, except that in the present context, the utility is assigned to consumption rather than net worth.

This being the case, it becomes necessary to immunise the cash flow required to support the subsistence consumption stream. The immunisation function becomes the more important because investment errors in old age can no longer be compensated by subsequent returns from human wealth. Moreover, borrowing requires debt servicing comfort to the lending institution that may not be available from the individual’s portfolio of financial wealth in all possible future states of the world. Finally, the required immunisation also has to encompass an unknown and variable maturity in the form of the remaining lifespan.

Even for the more affluent, it is useful to think of a layered approach to portfolio choice, executed in the decision phase. The base layer provides the income assurance for a minimal lifestyle over an uncertain time until death. Once this is met, a more adventurous stance is possible for the investment of the remaining wealth. Equity fund managers implicitly recognise the layered approach when they promote capital guaranteed funds, the base tranche of which is a long maturity zero coupon bond of impeccable credit but modest accumulation rate. The present proposals have some points of similarity with this idea, except that they apply to an entire stream of future income, and moreover allow for that stream to extend to an uncertain time to death.

Utility is generally also attached to the sum available on death to fulfil bequests, meet funerary expenses, or in some cases to continue support for dependents. However this need not imply that risk preferences are the same as between support while still alive and bequests on death. A more relaxed risk appetite might well hold as to the bequest motive. To be sure, there is often a desire to leave something to one's heirs, or not to saddle them with funerary expenses, or as endowments of one kind or another. Yet in many, if not most cases, there is a sense that what is left at death is a residual and lesser certainty imperatives apply. This suggests that enhanced security during life could be bought at the expense of certainty at death. Differential risk preferences are signalled by a willingness to pass income forward or backward through planning time. A good system should allow for a menu choice as to the extent of substitution between income while still alive and terminal value at death.

Turning to institutional provision, discussions of the problem of income assurance are commonly organised around the nature of the assurance, and the provider. The first is concerned with whether the income and/or bequest to be provided is defined benefit in nature, or whether it is unit linked, and therefore performance dependent. The second issue concerns who should provide the scheme, in particular as between the private sector and the government. It is beyond the scope of the present contribution to consider these issues in depth. But their interplay is important for what follows. If income certainty during life is of paramount importance, then defined benefits are called for. In addition, they should be realistic defined benefits and should be watertight as to risk, including both performance and the corporate failure of the provider. To achieve this agenda is the challenge.

2.2 Provider risk

Private sector annuities schemes are now generally unit linked in nature, with the payout tied in some way to what is available in the unit holder's account at the time. This immediately implies performance risk. The latter may arise from the fortunes of financial markets at the time, especially longer term trends not foreseen at inception of the contract. It can also arise from various elements of corporate risk, such as the departure of key investment personnel, to corporate takeovers by other institutions, or to funds outflow from the rise of new competitors or investment opportunities.

Also of concern is the issue of credit risk. The purchase of annuities typically involves the more upfront provision of funds to the institution (at a time when they are less needed by the purchaser) in exchange for a more back loaded provision of income benefits (at a time when they are needed). A credit exposure duration mismatch therefore arises, adverse to consumers (purchasers). If the credit of the institution deteriorates, this affects the pricing of its liabilities, even if the institution itself does not collapse. Deteriorating performance may or may not be readily detectable (it can often be disguised or obscured), especially if the institution does not actually issue debt and therefore have a publicly visible credit rating. But it is important to remember that the institution's liabilities are still there, in the form of any defined benefit elements, such as reversionary bonuses or guarantees. Nor is the credit exposure automatically extinguished by unit linked securitisation of the scheme, where this path is followed.

A third concern with private sector insurance-based schemes is the less than transparent pricing of their annuities. Many studies have pointed up large variations between institutions in this respect even where their asset composition is roughly the same. A systematic problem arises via the expected present value framework for pricing annuities, with informal adjustment for risk via more or less arbitrary risk premiums derived from CAPM or similar models. Practice in this respect contrasts with more formal market based pricing frameworks in finance theory and derivatives pricing, which are tied to observable interest rates and asset prices. A further problem is the variable overlay of management and insurance costs, which is the case of lifetime annuities can mount to 2.5% or more p.a., contrasting with the standardised and competitive margins or bid ask spreads of the traded market.

Public provision has some risks of its own, notably the policy inconsistency problem, wherein the government having announced one payout policy can subsequently change it, citing either a change in its economic circumstances or else priorities and contingencies unforeseen at the time. There is an agency problem here. Taxpayers might contribute now to infrastructure funding via general taxation revenues under the impression that they are contributing equity funding and will later share its revenue stream in the form of retirement income. But the government (as the agent for taxpayers) has a distributed agenda and the commitment is easily lost under the pressure of subsequent events and shifts of sentiment. Demographics – especially the baby boomers - have loomed large in the ability and continued willingness of governments to continue funding retirement schemes for the general population. Capacity to pay has become a risk to the provider in this case.

Nevertheless, risks of this kind not be allowed to obscure a potential role of the government in underpinning the certainty required of a retirement scheme, even if it does not provide the scheme itself. Governments of OECD countries have some key advantages in this respect: They are long lived, they have sovereign power and the ability to levy taxes , and they are capable of influencing the economic environment and hence the ultimate performance of financial assets. It is for such reasons that their debt typically carries the highest credit ratings in the economy. This can be used to underpin the provision of retirement income certainty, in its basic tranche. The use of dedicated long term asset vehicles (see below) would help to resolve agency problems by providing a formal corporate liability structure.

2.3 High grade debt as the physical anchor

To achieve the desired basic tranche of risk free income in the face of performance risk and time to death risk, and provider risk, there has to be an asset anchor. In this paper the anchor role is played by high grade long term debt. The derivative instruments or facilities to be described are all enhancements of such an underlying physical. The type and sources of such bonds are considered at more length in section IV.

Basic requirements are as follows.
(i) The bond should be of long duration. This is needed to create volatility in its price. In this connection, uncertainty in interest rates is seen as a positive source of value to be unlocked. The ideal instrument would be a perpetuity. Its duration is high, especially if the interest rate is low[3]. Hence its price fluctuates markedly in response to changes in interest rates and does so in just the right region, namely that where the coupon is itself low and in need of enhancement to become a realistic income instrument. Although governments these days do not generally issue perpetuities (UK consols are still available but only on the secondary market), it could be argued that they continue to have other obligations that amount to de facto perpetuities; thus little would change. However, even if this aspect is thought to be of concern, the enhancements to be described permit the bonds to be callable in one way or another, even if they are not formally described in such terms. There is indeed a natural call point, namely the point of death of the individual. In what follows it assumed for expositional convenience that the debt instrument is indeed a perpetuity, which simplifies pricing.

[3] For instance, the modified duration of a perpetuity under a flat term structure is $1/r$. Its convexity is also very high, meaning its value rises more when interest rates drop than it falls when interest rates rise, good for the holder. See Appendix A for a short elaboration.

(ii) The credit underpinning the bond should be extremely strong and perceived to remain so over all horizons. For this reason, the instrument will likely be a government or agency bond, although this does not rule out very high grade debt from other sources. Being able to ignore credit spreads does facilitate standardised pricing and benefit calculations.
(iii) The physical should itself be an income instrument. This will give the holder the facility to freely restructure the flow of coupons, including the ability to stay with the instrument in the form in which it was originally issued. Depending on the tax jurisdiction, there may also be taxation considerations; for instance there may be a requirement to view the instruments as intended to yield income rather than pure capital gain. There might also be reasons derived from credit arguments. An instrument such as a very long dated zero coupon bond might tempt the issuer to delay consideration of just how the debt would be serviced and hence create an adverse prospect for future repayment of principal.
(iv) The debt instrument should ideally be inflation protected, certainly as to coupons. A margin of choice exists as to whether or not to also protect the principal, if the chosen instrument has finite maturity. The latter will be long and the alternative always exists of sacrificing the principal protection to ensure a higher real coupon yield. The exposition that follows is in nominal terms, but can readily be adapted to handle inflation indexed debt.

III Lifestyle derivatives: structure

The underlying physical has one attractive feature, namely its credit rating and the comfort that this brings to retirees faced with the need to assure income in old age. But straight government bonds (for instance) also have some unattractive features: (a) In recent years the coupon has been low, driving investors to seek higher yields and therefore riskier investments, with inevitably adverse outcomes. (b) In some countries their maturity is not all that long (10 years or less) and therefore subject to coupon reinvestment risk when rolled over into new bonds. (c) Taxation provisions may be unfavourable. Ideally one would like to delay coupon receipt to later years at a lower marginal tax rate. (d) On death all remaining balances by default become part of the estate, leaving far more financial value than the individual might have chosen with the advantage of flexible allocation in advance. Given the inherent uncertainty of the time to death, or the onset of mental or physical incapacity, it is well nigh impossible to schedule with any confidence a rundown to match the precise pattern of needs in future years and at the also leave the individual with an optimal bequest.

The derivatives are designed to achieve payouts that more closely align the temporal pattern of effective coupons with the needs of retirement income. Given efficient markets, higher coupons over an indefinite period until death cannot be conjured out of nothing; something has to be given up. Higher lifetime coupons require compensatory sacrifice when death does eventually occur (or vice versa in rarer cases). However if this can be done, there are welfare gains.

The two generic types of enhancement to be discussed are as follows:

1. Lifetime annuity swaps, under which the individual swaps the perpetuity coupon for a higher coupon while still alive together with a compensating sacrifice of the value of the bond at the time of death. The proportion of terminal value to be sacrificed is set in advance by the individual. These can be piggybacked on forward start swaps, in which the individual with less need of income at the present time swaps the interim coupons for higher coupons at a designated future time.

2. Lifetime coupon options (LCO's), written by the perpetuity holder, with uncertain maturity set at the time of death. The strike price is chosen as the sacrifice parameter, in exchange for an option premium up front. The latter is then reinvested in the underlying perpetuity, and a second round option written on its price. The process is continued until convergence, in effect constituting an option in which the premium appears in the form of an enhanced coupon payable only during the life of the individual. The LCO refers to the entire compound option.

The swaps are not in themselves novel instruments, indeed they are simply lifetime annuities with a death benefit, the only special wrinkle being their uncertain maturity. They can be viewed as the debt market-based equivalent of many of the products offered by the insurance industry. However they are a good point to start in order to establish some early notation and conventions.

3.1 Lifetime annuity swaps

The physical bond is taken as a perpetuity. For expositional simplicity we will imagine that the term structure is flat and assumed to remain so, although the interest rate can itself vary up or down. Extension to non flat term structures are considered below in connection with options. Thus if the coupon on the bond is *y*, and the current (time 0) interest rate is *r*, then the price of the perpetuity $P_0 = \frac{y}{r}$.

Under the terms of the swap, the individual chooses a sum *A* to retain at death. Define the 'sacrifice ratio' as

$$\theta = 1 - \frac{A}{P_0}.$$

Thus $A = (1-\theta)P_0$, so that if θ = 75%, the individual elects to retain a bequest on death priced at 25% of the current value of the bond.

Remark: The sacrifice ratio could alternatively be defined in term of the current forward price, as having a reference to the future value of the bond. The two are equivalent for a perpetuity under a flat term structure. Assume provisionally that the time of death is T and let F_T be the time 0 forward price of the physical for maturity T. If the bond is a perpetuity and the term structure is flat, then the forward price is independent of T and in fact $F = P_0$, the current price of the bond, a convenient feature (see Appendix A).

The simplest way to price such a swap is to equate expected present values of the two sides, where the expectation is taken with respect to the probability distribution of the time of death. The process corresponds to expected present value annuity pricing, which is the predominant way of doing things in the insurance industry. There is an implicit assumption that the probability density of death is constant, not dependent on the economy, and interest rates in particular. A more precise specification is covered below in connection with options pricing, with adverse selection aspects also ignored for the moment.

Proposition 1

If the probability density of time to death is $\phi(T)$, then under expected annuity pricing with a flat term structure, the lifetime coupon obtainable is given by

(1) $$a(\theta) = rP_0(1+\theta\frac{d}{1-d}) = y(1+\theta\frac{d}{1-d}),$$

$$where\ d = \int_0^\infty e^{-rT}\phi(T)dT.$$

Proof: Take time as continuous (for later comparability with the options) so that r is the continuous time rate of interest. Suppose that the agreed lifetime coupon under the proposed swap is a. Conditional on death at future time T, the present value of the lifetime coupons is

$$PV|T = a\int_0^T e^{-rt}dt + Ae^{-rT} = \frac{a}{r}(1-e^{-rT}) + Ae^{-rT}.$$

Taking the expected value over different lifetimes (T) gives

$$E[PV] = \frac{a}{r}\int_0^\infty (1-e^{-rT})\phi(T)dT + A\int_0^\infty e^{-rT}\phi(T)dT.$$

Setting this equal to the value of the perpetuity P_0 and solving yields the required result (1). The same formula holds if we reinterpret time as discrete with r now referring to discrete time discounting.

□

According to equation (1), the apparent coupon is multiplied up by a factor that depends upon the sacrifice ratio and the density of time to death. Table 1 gives some indicative results derived from the official 2002 NZ life tables. The associated time to death density appears as figure 1. It is assumed that there is $1000 worth of base perpetuity income (this is y), and the continuous time interest rate is 6.6%, calibrated off NZ long term rates at the time. The numbers in brackets are the apparent lifetime yield relative to the bond price of $1000/0.066 = $15,151.52. There is no gain unless some of the bequest is given up, thereafter increasing sharply with the sacrifice ratio. Substantial enhancement is possible even at age 75.

Sacrifice on death	**Alive at 75**	**Alive at 80**
0%	1000 (6.6%)	1000 (6.6%)
25%	1289.36 (8.51%)	1412.09 (9.32%)
50%	1578.72 (10.42%)	1824.19 (12.04%)
75%	1868.08 (12.33%)	2236.28 (14.76%)
100%	2157.44 (14.24%)	2648.37 (17.48%)

Table 1: Lifetime swap coupons

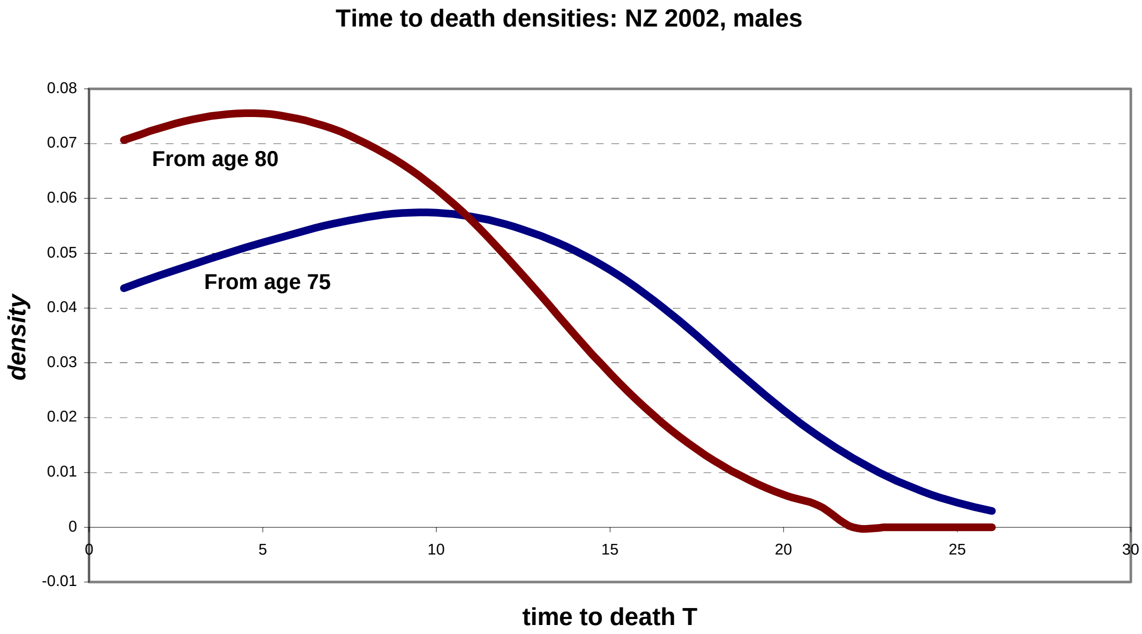

Figure 1: Conditional density functions for time to death.
(Original data source is official lifetables 2002, Statistics NZ)

Forward start facilities

Coupon gains from lifetime annuity swaps (or options as below) can be further enhanced by a choice of forward start facilities, executed at some prior date. These allow the individual to invest now but receive later. For example, the investor could invest a lump sum at age 65 in a structure that carries no coupons until age 75. At that point the accumulated value is taken out in a higher perpetual coupon. At some time during the coupon period the individual might elect to write a lifetime annuity swap, or LCO, further building on the higher coupons until the time of death.

The coupon gains from a forward start arrangement appear quite dramatic, even without further augmentation via lifetime annuity swaps or LCO's. For instance, if the current rate is 6.6% p.a. then with a 10 year forward start, the coupon at age 75 becomes $1934.79 instead of the original $1000. The investor has given up $1000 in the zero coupon period to be rewarded with $1934 in the coupon period until death, which could be further enhanced at the time via lifetime annuity swaps or LCO's.

Of course, there is no real financial gain in return terms. To see this, note that a forward start of this kind can be structured as a swap, in which the counterparty (say a bank) receives the straight perpetuity and an annual flow of $1000 starting immediately, while the retiree receives from the bank the perpetuity starting at age 75 with the enhanced coupon. The forward start coupon of $1934 is solved by equating the present values of the two sides to the swap, following standard market practice. Thus no new financial value has been created, indeed in practice there would be a slight loss in value to the individual due to swap spreads and other transactions costs. See also below on credit risk to the individual.

Gains in financial value could arise from tax considerations, depending upon whether accrual or cash rules were followed in assessing tax. It could be argued that this was an investment scheme under which money was put aside now to earn income later on. If taxation rules applied that are based on cash, rather than accrual or capital gains, then the taxation would be delayed until the coupons are actually received. By that time, the individual could be on a lower marginal rate of tax. The forward start instrument is therefore an intertemporal tax smoothing device. If this is the case, there would indeed be a tax source of financial value and the swap would have positive value.

Absent tax non- neutralities of this kind, there could still be a gain arising from the interplay of lifetime financial planning and risk. Suppose tax was levied on the accrual of gains during the zero coupon period. The individual would lose wealth and some utility from this, but would know that an assured (and enhanced) income is going to be available during the later coupon period. This assurance would diminish the risk premium applied to the coupon period, enabling the individual to undertake riskier investments at the outset. In other words, the forward start facility would lower the risk premium applicable to intertemporal portfolio optimisation. The gain from lifetime planning is taken up in section IV below.

3.2 Lifetime coupon options (LCO's)

The LCO's work as follows. At some point during the life of the basic physical bond, the investor can choose to sell (write) call options on the bond. The maturity of such (European) options is variable and is determined by the time of death. The strike price is set by the investor from bequest motives as above, balancing these up with his or her own likely needs as long as alive. The premium from the option is reinvested in the purchase of further units of the underlying debt physical, and further options written.

The enhancement to the basic coupon stems from two effects: (a) Even with a strike set at the current value of the bond (defined as zero sacrifice), the volatility of interest rates will ensure that the option has value, constituting a further income stream; (b) To the extent that the investor choose to sacrifice terminal value, this can be brought forward to further enhance the coupon, which will last as long as he or she is still living. The latter effect is achieved by investing the initial option premium in the same underlying physical bond, then writing a further call option on the additional component, and so on until convergence. The LCO refers to the complete arrangement.

This option can be priced under various assumptions about the probability density of time to of death and the way that this is related to the economy. The simplest assumption is that a specific individual's time to death bears no relationship to the state of the economy between now and then, or more precisely that the economy is not affected by the morbidity of its agents. In that case, one can price the option knowing the volatility of the interest rate and the density of time to death. More technical aspects of the above assumption are covered in Appendix B.

As with the annuity swaps we make the simplifying assumption that the term structure is currently flat (see below for remarks on non flat structures). The forward interest rate volatility is denoted σ. It is assumed to remain constant over time, and the same for any forward maturity (e.g. the term structure remains flat, in which case σ just the volatility of the physical perpetuity price). For any given maturity T we assume that the option is priced off the Black (1976) forward price model, which uses the forward price as basic input (also see Barone-Adesi and Whaley (1987) or Briys et al 1998). As earlier pointed out, this is just the same as the current spot price for the perpetuity. The following result prices the compound death option (i.e. with reinvestment at each stage).

It is convenient in this context to define the sacrifice ratio θ in terms of the strike price of the option. Thus $X = (1-\theta)P_0$, and the ratio θ of the strike to the value of the physical remains the same for each reinvestment stage.

Proposition 2

Suppose the sacrifice ratio θ is the same for the strike prices set for options written at each reinvestment stage of the option premiums. Given any given maturity T, define Black pricing factors as follows:

$$\lambda(T,\theta) = N(d_1(T,\theta)) - (1-\theta)N(d_2(T,\theta)]$$

where

$$d_1(T,\theta) = \frac{1}{\sigma\sqrt{T}}[ln(\frac{1}{1-\theta}) + 0.5\sigma^2 T]$$

$$d_2(T,\theta) = d_1(T,\theta) - \sigma\sqrt{T}.$$

Let the expected value of the Black pricing factors taken over time to death be

$$\lambda(\theta) = \int_0^\infty \lambda(T,\theta)\phi(T)dT.$$

Then the compound death option is equivalent to one with strike value

$$(2) \quad X^*(\theta) = \frac{(1-\theta)P_0}{1-\lambda(\theta)},$$

written on a perpetuity paying a coupon

$$(3) \quad b(\theta) = \frac{y}{1-\lambda(\theta)}$$

with market value $P_0^*(\theta) = \dfrac{P_0}{1-\lambda(\theta)}$.

Proof: see Appendix C.

As with the annuity swaps, there is an enhancement of the coupon while still alive. In effect the individual ends up owning $1/[1-\lambda(\theta)]$ perpetuities instead of just one. This is achieved by giving up the gains if the value of the perpetuity at death exceeds the strike price. This feature aligns with the trade off between certainty of income while still alive and a lesser or more variable bequest motive on death.

Table 2 present some results with r = 6.6% and a volatility of 18%, calibrated off recent NZ 10 year bond yields. At zero sacrifice levels, i.e. the strike set equal to the forward value of the bond, the coupon enhancement is superior to the lifetime annuity (see table 1). The individual has gained from the volatility, giving up any potential gain in the price at death. Note also that the individual remains exposed to any shortfall in the terminal bond value relative to the strike price (see below).

age 75				age 80			
theta	total coupon	yield	total strike	theta	total coupon	yield	total strike
0	1,115.80	7.36%	16,896.67	0	1,118.04	7.38%	16,940.00
0.1	1,146.24	7.57%	15,630.47	0.1	1,155.91	7.63%	15,762.34
0.2	1,186.80	7.83%	14,385.47	0.2	1,206.80	7.96%	14,627.81
0.3	1,238.71	8.18%	13,137.84	0.3	1,273.49	8.41%	13,506.65
0.4	1,303.84	8.61%	11,853.11	0.4	1,358.95	8.97%	12,354.05
0.5	1,384.47	9.14%	10,488.42	0.5	1,467.01	9.68%	11,113.73
0.6	1,483.51	9.79%	8,990.95	0.6	1,603.00	10.58%	9,715.12
0.7	1,604.64	10.59%	7,293.80	0.7	1,774.38	11.71%	8,065.35
0.8	1,752.58	11.57%	5,310.84	0.8	1,992.15	13.15%	6,036.82
0.9	1,933.67	12.76%	2,929.80	0.9	2,273.61	15.01%	3,444.86
1	2,157.44	14.24%	0	1	2,648.37	17.48%	0

Table 2: Trade offs for LCO's

Figure 2 below compares the death option with the lifetime annuity swap in terms of the trade off between total coupon achieved and the value at death. To generate this trade off we vary the sacrifice ratio between 0% and 100% in each case, so that the latter becomes the generator of the trade off curves. There is little difference for high sacrifice levels. The latter means that the options are well out of the money and virtually equivalent to the swaps. Hence the horizontal axis has been truncated to omit higher coupon rates received as a result of high sacrifice rates.

A difference does appear for low sacrifice levels with higher lifetime coupons achievable in the options. However, the superiority of the options to the swap is more apparent than real, for it should be recalled that there is residual risk in the option case where the terminal bond price falls below the strike price and this is borne by the purchaser. If the individual dies at age T, then the terminal payoff is equal to $min\,[P_T^*(\theta), X^*(\theta)]$. The expected value, taken over the random time to death and the bond price, is certainly less than the strike price and would plot somewhere between the two curves of figure 2, depending on particular assumptions about the drift of the physical bond price. In figure 2 we have also plotted the two sigma (approx 5%) terminal value at risk (*TVaR*) for the lifetime coupon option, assuming that the term structure remains flat and there is zero drift in bond prices. At very high sacrifice ratios, the strike price is being set at a large discount to the current bond value and there is lesser risk of further loss at maturity. At lower sacrifice ratios, the strike is priced at or near to the money and there is appreciable risk to the terminal value.

In an efficient market, there should be no net pricing advantage of the death swap over the death option. A more appropriate interpretation is that the annuity swap represents a market certainty equivalent to the option. With reference to figure 2, points B and C are certainty equivalent. Points A and B are not, for A carries no risk while B does, namely the bequest shortfall should the physical price fall below the strike price at maturity. Thus if the individual specifies a sacrifice ratio to give a coupon of $1200 in each case, then the expected terminal payout under the LCO, taken over all possible times to death, is less than $14,000, which is the nominal strike on the option. The certainty equivalent terminal payout is given by \$12,511.55, which is the terminal payout for the annuity swap that yields the same coupon.

It is worth remarking that the certainty equivalence involved is that of the representative market agent. Point B could still judged as be superior to C where the individual was less risk averse than the market as a whole. As earlier remarked, this might happen because of the particular role played in retirement planning by the bequest, namely as a less sensitive item than is income while the individual is still alive. Thus the individual's risk premium in this respect could be less than that for the market as whole. The relationship with the remainder of the retiree's investment portfolio might also be relevant. The risk to the option arises when the time of death physical price drops below the strike price. Such a contingency suggests higher interest rates but by the same token, this might benefit earnings to that point on the rest of the portfolio, cushioning the risk of the bequest loss. Thus non neutral risk considerations arising from lifetime risk profiling could lead to preferences for the option.

The LCO's might also have some psychological advantages over the straight annuity swaps. Suppose the latter were chosen, with a non-zero sacrifice specified. The individual would know that if he or she died early, the sacrifice of terminal value would have been expensively bought, and a source of annoyance to the estate beneficiaries. The regret effect is less a feature of the option, where shortfalls in terminal value may well occur but are more a result of market forces on the price of the underlying physical, and outside the control of the individual.

Finally, the basic form of the compound solution as in Proposition 2 remains intact if the term structure is not flat. What happens in this case is that the current forward rates differ according to maturity. Given any maturity T, the pricing factor can be written as $\lambda(T,\theta,\frac{F_T}{P_0})$ and $\lambda(\theta)$ is the expectation of this over the time to death T, as in Proposition 2.

Trade off between total strike and total coupon: Age 75

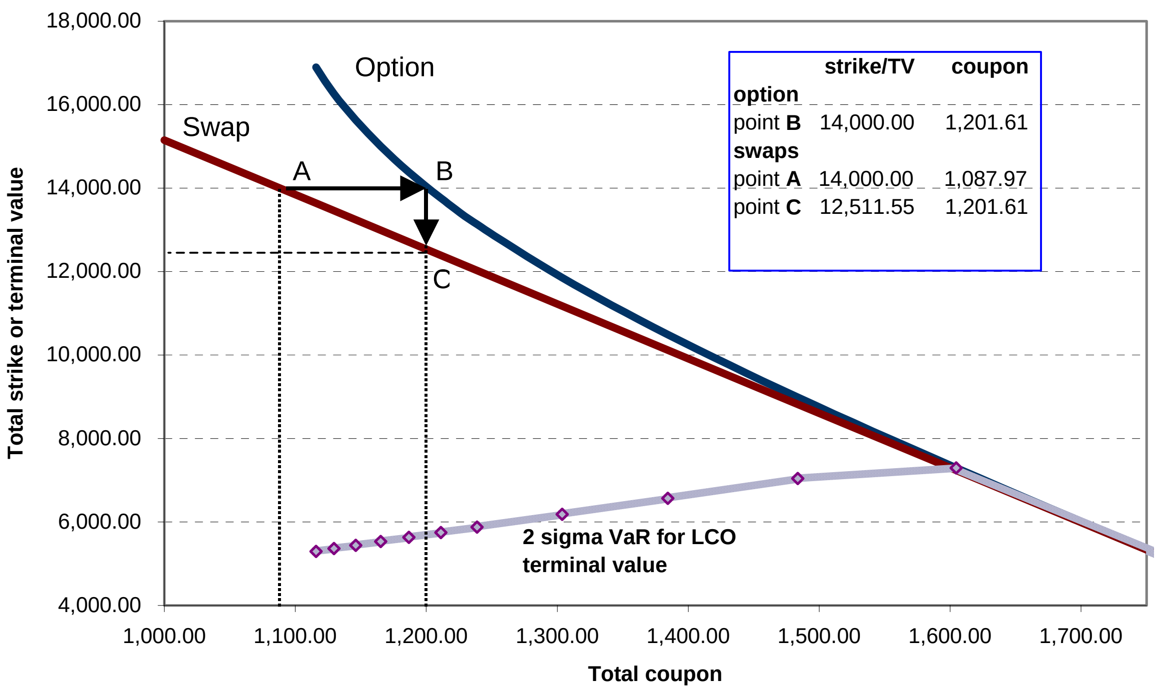

Figure 2: Market equivalences between LCO's and annuity swaps

3.3 Directional risk shifting: calls versus puts

The effect of the LCO's is to shift risk forward in time on to the terminal value, so that the terminal value or bequest becomes uncertain downwards. This enables the individual to derive income from writing the corresponding call premium. A more common arrangement in the insurance industry is to set a floor to the terminal value, so that the individual is guaranteed a base amount and is paid more if the balance of the account at death is worth more. In option terminology, this would amount to the individual purchasing a put option on the terminal value. Doing this means that he or she is giving up existing income while still alive. This amounts to assuming that the individual is relatively less risk averse for lifetime income and more risk averse for terminal value. This could be justified in certain cases, e.g. where a dependent will remain present after the death of the retiree.

If both calls are sold and puts purchased, this amounts to a straight annuity swap. Appendix D shows that the lifetime annuity swap payoffs can be nested within the option framework by imagining that the individual both sells a call and buys a put, at the same strike price X = A. The total effect is to secure a certain payoff in any state of the world at death. The coupon achievable reduces to that of the annuity swap, demonstrating the consistency of pricing between the two structures.

3.4 Enhancements: credit risks, and institutional provision

A government instrumentality or another high grade issuer could write the core perpetuity or long term liability. Who would supply the enhancements, and is there is a role for the private sector? The two kinds of enhancements proposed differ materially in this respect according to who bears the credit risk.

LCO's are written by the individual and the purchaser pays the premium up front. The individual has no intrinsic exposure to the purchaser with respect to future cash flows. The purchaser is exposed only to the extent that the underlying core perpetuity (written by the government) is not deliverable for some reason upon death of the writer. However this is a minor risk easily resolved by an escrow or senior claim arrangement on the underlying physical.

Thus there is no reason why the lifetime coupon options could not be arranged and purchased by banks, insurance companies or fund managers. Indeed there could be tranches of LCO's according to their expected maturity, which would thereby carry different degrees of interest rate risk. The earnings on such instruments could derive from fees, which would be set competitively, or from different views about the options volatility or pricing, in the normal way.

The forward start feature could be structured as a swap, as earlier outlined. Banks would like such a swap because the individual pays up front, whereas the bank pays in arrears at the end of the zero coupon period. All that would be necessary is some institutional arrangement to bundle the multiplicity of small transactions into economic parcels.

On the other hand, individuals are now exposed to the bank. They have passed the early coupons through to the bank, but as retirees are to receive on the swap at a later date and effectively do so in perpetuity (especially if the death option is sold). Thus much depends upon confidence in the longevity of the banks or on the seniority of the swap liability should there be trouble. It could be pointed out that banks, especially clearing banks, are more closely supervised and therefore the more unlikely to fail in the first place, though the same security might not necessarily apply if the functions in question were to be undertaken by a special purpose subsidiary.
If this is thought to be a problem, an obvious solution is to have both the core perpetuity and the forward start written by the government instrumentality. However this does increase the complexity burden for the latter – it is in many respects cleaner to have the government confine its activities to the issue of the core debt instrument and leave it to the market to devise enhancements, just as happens now with zero coupon bonds and coupon strips.

Lifetime annuities structured as a swap create a duration mismatch that is potentially adverse to the creating institution. From their point of view, the payments they are making are limited by the time to death of the individual and the need to pay a certain sum on death (A). But on the asset side, they are receiving a perpetuity. Corresponding to table 1 above, table 3 shows the durations for a lifetime annuity swap at various sacrifice ratios. They range from 4- 7 years. Compared with the asset duration of 16.15 years for the perpetuity at 6.6%, there is evidently a substantial duration mismatch for the writing institution. The institution is exposed to the reinvestment risk of the coupons it receives on the perpetuity, as well as to the capital values of the perpetuity down the track that might be used to meet any bequest motives. Particularly damaging would be an initial fall in interest rates (penalising the early reinvestment of coupons) and a rise later on (hampering the cover of any terminal bequest motive). To handle this risk it will be necessary for the institution to shorten the duration of its assets by means of further swaps or physical transactions.

Sacrifice on death	Alive at 75	Alive at 80
0%	7.259	5.864
25%	6.916	5.600
50%	6.573	5.336
75%	6.229	5.072
100%	5.886	4.806

Table 3: Annuity durations

3.4 Adverse selection

The problem of adverse selection is well known in the annuities trade. Thus an 80-year old would receive the higher perpetuity payouts designated for this age. But his private information might well be that his health and life expectancy are more like those of a 75-year old. He will therefore end up receiving the higher 80-year old rewards for a longer period of time. Conversely, a 75 year old whose expectancy is less than the norm, and privately suspects it, will feel less inclined to contract under an annuity arrangement of any kind.

The adverse selection arises because the population of purchasers is not the same as the general population in a material aspect. Most solutions to this adverse selection problem impose conservative probabilities, relative to national life tables. So the death options or annuities for an 80-year old would be priced according to the 75 year old probabilities, so the payout would be the lower payout of the first column of the table. A similar problem arises where the options cannot be priced because the time to death cannot be perfectly hedged. In both cases there is mispricing risk for the option.

The issue arises as to whether the possible enhancements are respectively neutral as to the incidence of adverse selection. Table 2 above showed that the lifetime coupons achieved with the LCO arrangement are not much affected by the age at which the options are purchased. Only at higher sacrifice ratios, where they become priced like an annuity, does this become the case. This suggests that when taken near the money, the LCO's are less exposed to the adverse selection problem.

IV Debt as the physical underpinning

The defined benefit schemes earlier described all arise as enhancements of an underpinning debt issue as the physical. The debt has to be very long term and it has to have very high credit rating. Whose debt?

Some private sector corporates issue very long dated debt in the form of bonds, debentures or more rarely, senior notes. Maturities of 20-30 years are reasonably common, but in recent years there have been some issues even longer. IBM and Apache both issued 100 year bonds (century bonds) in 1996, while Walt Disney's 100-year issue in 1993 quickly became part of financial folklore as 'sleeping beauty' bonds, taking the fizz out of Coca Cola's 100-year issue in the same year. These issues represented something of a throwback to practice at the turn of the century; in the 1890's, two thirds of bonds issued bore either 50 or 100-year maturities.

Private sector debt of this kind has the advantage of a higher yield, but has two disadvantages. First the necessary degree of liquidity may be lacking. Second, credit ratings always refer to the here and now. A high current credit rating may have little reference to what can happen down the track following major change in product markets, technology or in corporate control. Doubts may still be felt as to whether the debt will still be viable or marketable in 20 years time. Moreover, the rating agencies can simply make mistakes; the underlying information may have been window dressed or adverse exposures hidden in special purpose vehicles. Clearing bank debt might be relatively immune to such an information deficiency, as they typically receive much greater supervision and oversight from the central bank in the pursuit of systemic safety. There is no intrinsic reason why banks could not offer debt of any maturity, the only effective requirement being the existence of a swap market based on counterparty corporate and government debt of the required maturity. Supervision of insurance company debt or other liabilities is less rigorous, notwithstanding a role of this kind by blanket supervisory agencies such as the FSA in the UK or APRA in Australia.

Some international agencies issue very long dated debt, notably the World Bank and the Asian Development Bank (ADB). The former issues out to 50 years and the ADB to 30 years. Both are agencies of the UN, and their debt could therefore be considered to derive comfort from the implied backing of the OECD nations. Although residual doubts remain about the commitment of the US as the major contributor, the rise of China and India may in the future years shore up the financial support base. The World Bank and the ADB have AAA (S&P) or equivalent credit ratings, the highest possible. The debt of such institutions remains a possible underpinning. The drawbacks might be liquidity (long term issues are only one of many maturity bands in many currencies by these organisations), and the absence of any commitment to continue to issue in these bands.

Commitment to issue implies recognition that the debt in question is being used to back a retirement scheme, as well as the more usual financial objectives. In other words, the issue process now has a multi item agenda. It cannot be expected that the World Bank should subscribe to supporting the retirement planning of OECD countries as being a proper object. That is more to be expected of a national issuer. Even here, there may be insufficient recognition of the externalities of a public debt programme. Thus in 2001, the US Treasury announced that it was ceasing to issue 30 year bonds, citing some rather ill defined ‘financial grounds’. The move was not popular with the markets, not least the CBOT as provider of the 30 year T bond future and options contracts.

The view taken here is that the long term debt of national governments or their agencies is the most natural underpinning for certainty in private savings-based retirement income. At first sight, issue of more long term public debt is an unpopular move especially among conservative commentators, who appear at times to forget the constructive role of debt in private enterprise. However, it need not be. The central issue is what is done with the proceeds, the asset cover, the risks involved, and the intergenerational debt servicing burden. Moreover, negative current attitudes to public debt generally fail to consider its beneficial aspects, not least in helping to solve some of the economic discipline problems that are otherwise commonly associated with government deficits. In this respect, an advantage of debt is that it is a first call on the government’s assets. Reneging or defaulting become much less possible and carries much more far reaching penalties for the government. In other words, one is imposing a financial markets discipline on the government as our collective agent, in much the same way suggested by Jensen (1986) for private sector debt. The same discipline forestalls the policy inconsistency problem earlier referred to. Once the debt is in the hands of the public, the government cannot change its terms; the holders have an asset secure against any changes in policy. The government has too much to lose.

Debt enhancements can be designed to assist the government in its own future obligations in adverse states of the world. For example, LCO’s can be regarded as callable bonds. When times are bad, interest rates are often low (Japan in a recent example.). Exercise of the implied call option at a pre-set strike price will result in a gain to the government. The mark to market loss to the individual’s bequest motive from such an event might nevertheless be consistent with consumer tolerance.

Finally it could be pointed out that even if the government does not offer formal retirement schemes, it nonetheless has a liability to fund those people who for one reason of another do not, or cannot, make their own provision. The use of public debt to underpin such a scheme can be regarded as replacing an implicit government liability with an explicit one. In the process has desirable social outcomes such as the increased savings rate and increased self reliance in providing for one’s own retirement.

In summary, the use of debt enhanced in the way to be suggested also becomes a way to ensure the certainty of defined benefits, as against qualified or nebulous promises that may be dependent on the performance of a fund manager, the national economy or government policy.

The asset cover

Responsible governments should not issue debt willy nilly, and nor should they assume liabilities that cannot be funded in one way or another. The latter aspect has made governments wary of accumulating liabilities arising from retirement financing. A risk assessment entails the following:

(a) The nature of the demographics. The babyboomer generation will make huge calls on any retirement funding scheme, whether financed out of general government revenue or otherwise. If in addition the population growth from birth rate or immigration sources is projected to be low, then there is a demographic squeeze.

(b) The growth in servicing capacity via economic growth, national income and the tax take. Growth pessimism is, in turn, linked to the demographics; for older workers are less capable of creating and driving economic growth and more of a consumption burden in the need to finance it. An unacceptable shifting of the burden on to the future generation is the result.

Debt used to back retirement schemes could always be issued as part of the general government funding programme, backed by servicing out of general revenue. A more transparent arrangement, however, is to duration-match the debt with designated asset cover arising from a specific economic asset base. These could be described as long term asset vehicles (LTAV). They are not set up just for the purpose of providing debt, but their existence can be exploited in a synergistic fashion to underpin a debt programme of the required kind. Depending on the nature of the assets, the asset cover could be used to back debenture issues, the collateral servicing to act as an alternative security to a general government guarantee.

The type of asset cover depends on considerations like (a) and (b) above. The pessimistic scenario is where both are present – i.e. an aging population and poor economic growth. Thus some governments, notably Norway, Ireland, New Zealand and the State of Alaska, have established partly or fully funded retirement schemes. These dedicated funds are supported as to inflow of investment monies via taxation or similar levies arising from current budget surpluses. The funds invest in a variety of assets, including equities, bonds, and real estate, both home and abroad. A government instrumentality of this kind could supplement its inflow by issuing bonds to the general public. The maturity of these bonds could be tailored to the assets of the fund as a whole, which in any case would be fairly long, if not indefinite. The investment earnings of the fund underpin the resulting debt servicing, with the government as guarantor of the fund assuming any residual liability. The assets of the fund could alternatively be used to provide collateral, as earlier noted in connection with debenture bonds.

If the fund is reasonably successful in riding the risk premium on equities and similar assets, it should generate a surplus or margin - indeed the investment target is commonly to exceed the return on government bonds. The investment margin generated by the issue of the new bonds is then available for consolidation with the fund’s own general revenue and can be used for future distribution according to its established policy guidelines, including the social goal of subsidising retirement income for those who cannot be counted on to make their own provision. Finally, the ability of the public to commit discretionary amounts for longer term goals would enhance the savings performance of the economy as a whole, an aspect that has been of concern in most OECD countries.

A second type of LTAV might take the form of a public sector infrastructure fund, perhaps with the creation of a public infrastructure financing corporation as a special purpose vehicle of the government. If such investment is to be worthwhile it should generate economic returns either explicitly via tolls or other use charges, or implicitly via increased incomes and taxation to general government revenue. In the latter aspect it has the advantage that the expected earnings stream encompasses externalities and is therefore not limited to income arising from the project itself. Moreover, these social returns should meet or exceed a benchmark provided by the government bond rate. Infrastructure is by nature a long term investment, matching the required maturity of the bonds. On the other hand, the risks of the public sector infrastructure LTAV would also span pork barrelling and other control or management risks as well as commercial risk arising from the projects themselves. Its debt would continue to be underpinned by a government guarantee.

A third type of LTAV is the government agency, such as Fannie Mae or Freddie Mac in the U.S., which have a dedicated type of long duration asset backing and large scale issue programs to finance it. See Fabozzi (2002, ch. 5) for a review of US federal agency securities. State owned enterprises for power generation etc. would also be of this kind. Another interesting possibility is the constructive use of government guarantees on debt issued by the private sector, e.g. to finance student loans. Debt of this kind could be packaged together by a special purpose entity.

In all cases, the use of publicly owned LTAV's imposes a formal corporate structure on the assets and liabilities. Retirement income can be supported out of both income from tax-originated equity and from the coupons on the debt. This helps to resolve the agency problems otherwise present in public funding of retirement schemes.

V Concluding remarks

Lifecycle planning and portfolio layering

In theory, instruments such as lifecycle options or annuities can be encompassed in standard intertemporal models of investment and consumption. Thus the Merton model and its extensions (e.g. Duffie 1996 ch.9) allow for a bequest motive on death as well as an ongoing consumption stream while still living. Expected intertemporal utility is maximised subject to a budget constraint that all trading strategies must be self financing. However, there are some complicating factors: (a) Any intertemporal trading strategy must allow for a random time to death. The same problem creates a credit risk that limits the supply of debt and it would be more realistic to suppose that debt was simply unavailable past a certain age. This would mean that at any time, consumption is limited by the available financial wealth, i.e. a cash in advance constraint at each point in time. (b) Even absent transactions costs, people simply do not have the time, expertise or resources to implement an appropriate trading strategy, including any attempt to replicate the payoffs of options or swaps from more basic securities. The situation is analogous with the huge volume of swaps and options created for and used by the corporate sector in their risk management and funding processes. Provision of death options and swaps would effectively complete market instrument availability for the personal sector just as they do for the corporate sector.

The role of such instruments would be to simplify lifetime portfolio planning, even if this remains suboptimal in theory – a case of 'satisficing' rather than true optimisation. There are now two sources of risk to handle, namely investment risk and time to death risk. The coupon stream available from derivatives backed by perpetuities of strong credit helps to assure the basic immunisation tranche of future cash flow in old age. It remains compatible with an overlay of more aggressive investments that may carry higher yields at the cost of more risk. If financial circumstances allow it, the portfolio can be structured as a layered investment mix of successively higher risk classes. Only if the latter carried substantial short sell position would the immunisation function of the primary layer be endangered and few, if any, older people would have the credit or courage to do this. Derivatives like LCO's or lifetime annuity swaps are open ended as to the time of death, and hence cover lifespan risk arising from this source.

The lifecycle planning prescription (section 2.1) calls for an initial accumulation of wealth in via any preferred mix of risky assets; the aim is simply as much accumulation as is reasonably consistent with desired consumption in this phase. By the time the retirement review and decision phase is reached, the individual will have decided what level of old age consumption is desirable and feasible, setting immunisation levels for future cash flow. The basic tranche of cash flow needed to support this can now be locked in by means of forward start facilities, or by immediately entering into annuity or options arrangements if the desired conversion point is seen as sooner rather than later.

The choice between LCO's and lifetime annuity swaps is governed by preferences as between ongoing consumption risk and the bequest motive. Many people would have a more relaxed attitude to the latter, with lower risk aversion parameters, and might therefore choose options, which trade off terminal risk for a higher assured coupon while still alive. Circumstances such as the continued need to support dependents might force others to a more risk averse stance for bequests, and they would be more likely to elect the certainty of annuity swaps.

Some public finance aspects

Publicly mandated retirement provision has typically been viewed as a straight choice between public funded schemes with taxation as the ultimate source of funds, and privately sourced provision with or without tax incentives. The former entails a moral delivery obligation on the part of the government, with risks of policy inconsistency. The latter imposes default and investment performance risk on both the annuity purchasers and the government as the welfare support of last resort, as well as considerable deadweight costs in the form of expensive management fees, always a first call on distributions whether good or bad. Neither arrangement is fully transparent as to where the risk is, who bears it, and whether it has been priced or otherwise factored into the decision to legislate for that type of retirement support.

The use of public long term asset vehicles represents a more corporate model, with the same transparency of ownership and formal commitments to distribute the resulting free cash flow. As with a private sector corporation, LTAV funding could be via equity both debt and equity. Equity would be provided from tax income, either as general revenue or from special purpose taxation. Relying solely on equity is a limitation, just as it would be for most private sector corporations. This would be a valid model only when the government has set its face against any form of private ownership of the cash flows of the LTAV, perhaps on equity grounds. However such concerns can be solved by the use of means testing and similar devices, which amounts to distributing the equity dividends to those whose circumstances most demand it.

The debt of the LTAV is now privately owned but the equity is publicly owned, so that what is entailed is a mixed private-public provision for assured retirement income.

The issue of relatively large amounts of new public debt has potential consequences for interest rates (such as ‘crowding out’ private sector debt) and other externalities for the economy. To the extent that the debt has a captive market, namely of future retirees, the interest consequences need not be a worry, especially if the national financial market is linked to the global markets, with an almost limitless appetite for high grade debt. Indeed it could argued that some OEDC countries have in recent years instituted debt redemption programmes that have actually penalised the efficient operation of financial markets, resulting in a shortage at the long end and contributing to an inverse term structure (New Zealand and Norway have been instances). In such circumstances, the effect of the new debt programmes could see a diversion of funds from the housing or stock markets and other competing assets. In itself, that may not be a bad thing, helping to curb speculative excesses in the latter, created in part by a necessity to build up cushioning capital for old age.

Moreover, if interest rates are already high, for foreign exchange risk premiums or other reasons, then the use of public debt to back retirement schemes becomes a constructive exploitation of what is otherwise an unavoidable evil. Small open economies commonly fall into this category, and so might larger economies from time to time as a result of public funding contingencies.

Finally, the problem of interpersonal equity arises in any mix of public and private retirement provision. Not everyone will have the wealth necessary to purchase the underlying debt instruments as a basis for the enhancement process. For such people, government support will continue to be necessary. As a consequence, a potential inequity arises as between those who have made their own provision and those who have not. Devices such as income or wealth testing for government or agency benefits only emphasise the inequity and the implied penalty for prudence. One solution is simply to make the same distribution to everyone, whether from general government revenue or from LTAV free cash flow to equity. A more economical solution might be to allow tax deductions or deferrals during the life of the recipient for the coupons received from the LCO’s or annuity swaps, i.e. those based on the designated debt instrument. A distinction could be drawn between the coupon and bequest elements in this respect, with the latter not eligible for any tax benefit to the estate.

Appendices

A: Forward price for a perpetuity

Suppose at time 0 the term structure is flat at rate r_0 .The forward price for a perpetuity is the same for any maturity T, and is given by the current price of the perpetuity.

Proof

For any fixed maturity T the coupons *y* earned between time 0 and time T are treated as a negative carry cost. Thus the forward price is given by

$$F = [\frac{y}{r_0} - \int_0^T e^{-r_0 t} y dt)] e^{r_0 T}$$

$$= \frac{y}{r_0} = P_0.$$

□

Note that if the term structure is always flat, the volatility of the forward price will just be the same as the volatility of the interest rate.

Duration and convexity properties

Let P be the price of the perpetuity, so $P = \frac{1}{r}$, where r is the current rate of interest (flat term structure). Modified Macaulay duration is $-\frac{1}{P}\frac{dP}{dr} = \frac{1}{r}$ and modified convexity is $\frac{1}{P}\frac{d^2P}{dr^2} = \frac{2}{r^2}$. The corresponding unmodified duration (or just 'duration') and unmodified convexity are respectively given by $d = (1+r)\frac{1}{P}\frac{dP}{dr} = \frac{1+r}{r}$ and $(1+r)^2\frac{1}{P}\frac{d^2P}{dr^2} = 2d^2$.

Relative convexity is a normalisation that expresses the convexity relative to that of a zero coupon bond with the same duration:

$$\textit{Relative convexity} = \frac{convexity}{d \times (1+d)}.$$

Thus relative convexity for the perpetuity is equal to $2\frac{d}{d+1} = 1 + \frac{1}{1+2r}$. At low interest rates this is almost twice the convexity for the zero coupon bond of the same duration.

B. Pricing the core bond option

Denote by $\Omega = \{w\}$ denote an event space for the economy. Let $\tilde{T}(w)$ be a random stopping time corresponding to the individual's time to death. The price of the bond at death can be written as $P(w, \tilde{T}(w))$ or just $P(w, \tilde{T})$. If X is the strike price, the payoff to the call option is

$$(P(w, \tilde{T}) - X)_+ = max[P(w, \tilde{T}) - X, 0].$$

Let ξ_t be the associated risk premium process in the economy and let r_t be the spot interest rate at any time t. Using standard equivalent martingale arguments, the option premium at time $t = 0$ can be written as

$$E_0[\xi(w, \tilde{T}) e^{-\int_0^{\tilde{T}} r_t(w) dt} (P(w, \tilde{T}) - X)_+].$$

The current starting point time ($t = 0$) will be understood in what follows and the expectational subscript *0* dropped.

Now assume that for any fixed time T, the state history of the economy up that any time does not depend upon whether our individual is alive or dead by then (the insignificant agent assumption). This is a strong path independence condition. Thus if $\Im_t$ is the natural filtration generated by the set of events $\{ w : \tilde{T}(w) \le t \}$ then for any fixed time T, the random variables $P(w,T)$, $\xi(w,T)$, and $B(w,T) = e^{-\int_0^T r_t(w)dt}$ are jointly independent with respect to $\Im_T$.

Using the iterated expectation.

$$E[\xi(w,\tilde{T})B(w,\tilde{T})[P(w,\tilde{T}) - X)_+] = E_{\tilde{T}}[E[\xi(w,\tilde{T})B(w,\tilde{T})(P(w,\tilde{T}) - X)_+]|\tilde{T}]$$

Given the strong independence assumption, the inner expectation is identical with the price $c(T)$ of an option with fixed expiry time $T = \tilde{T}$. Let the density of the time to death, given that the individual has survived to age a, be denoted $\phi(T;a)$. Then the price of the death option is given by

(Ai) $$c(a) = \int_0^\infty c(T)\phi(T;a)dT .$$

In what follows, the age at purchase will be suppressed, so we write $c = c(a)$, similarly just $\phi(T)$, with the age dependence being understood.

Remark: The above approach technically requires that each individual's time to death can be continually hedged, which is obviously unrealistic, with market completeness problems. Thus it is better suited to pricing a large portfolio of such instruments. Even here, it remains susceptible to misspecification or changes in the survival probabilities. Hence a cautious approach is called for- see the remarks in section 3.4.

C. Proof of proposition 2

If the forward price for time T maturity is F, the option strike price is X, and the volatility of the forward price is σ, then the Black price is given by

(Aii) $$c(T) = e^{-r_0 T}[FN(d_1) - XN(d_2)]; \text{ with}$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}[ln(\frac{F}{X}) + 0.5\sigma^2 T];$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad .$$

For a perpetuity (given the flat term structure), $F = P_0$ the current price, and we set $\theta = 1 - X/F$; or for a given sacrifice ratio θ, the strike price will be $X = (1-\theta) F = (1-\theta)P_0$. This gives the value of the call option as

$$(Aiii)\qquad c(T,\theta) = P_0\,\lambda(T,\theta),$$

$$\text{with}\qquad \lambda(T,\theta) = e^{-r_0 T}[N(d_{1T}) - (1-\theta)N(d_{2T})];$$

$$d_{1T} = \frac{1}{\sigma\sqrt{T}}[\ln(\frac{1}{1-\theta}) + 0.5\sigma^2 T];$$

$$d_{2T} = d_1 - \sigma\sqrt{T}\quad .$$

Taking expectations with respect to the time of death density $\phi(T)$ gives the value of the first round option as $c_1 = P_0\lambda$, where for brevity in what follows, we write λ(θ) = λ with the θ understood. Note that $|\lambda| < 1$.

The first round call premium is reinvested in a new perpetuity to give a second round coupon of $rc_1 = y\lambda$. A new call option is also written on the capital value c_1. This is priced as above with the same sacrifice ratio θ. It will have strike price equal to $(1-\theta)c_1 = (1-\theta)\lambda P_0$. The value of the second round option will be $c_2 = c_1\lambda = \lambda^2 P_0$. This invested in a new perpetuity to give a coupon of $r\lambda^2 P_0 = \lambda^2 y$. The capital sum is invested in a third round option with a strike price of $(1-\theta)c_2 = (1-\theta)\lambda^2 P_0$.

The process is continued until convergence. The total coupon generated and available until death will be

$$y + y\lambda + y\lambda^2 + ... = \frac{y}{1-\lambda}.$$

The total value of perpetuities owned will be

$$P_0 + \lambda P_0 + \lambda^2 P_0 + ... = \frac{P_0}{1-\lambda};$$

and the total strike price value will be

$$(1-\theta)P_0 + (1-\theta)\lambda P_0 + (1-\theta)\lambda^2 P_0 + ... = \frac{(1-\theta)P_0}{1-\lambda}.$$

□

D: Pricing correspondences: lifetime annuity swaps and LCO's

Lifetime annuity swap payoffs can be nested within the option framework by imagining that the individual both sells a call and buys a put, at the same strike price X. The total effect is to secure a certain payoff X in any state of the world at death. The contingency table below illustrates, for a fixed time of death T; P_T is the price of the bond at that time.

	P_T < X	P_T > X
Sell call	0	X-P_T
Buy put	X-P_T	0
Physical	P_T	P_T
Total	X	X

This fact can be used to derive the swap coupon as an alternative to the expected PV approach. Using put-call parity in the Black model, and recollecting that $F = P_0$, the net premium income from the first round options, for a given time to death T, is obtained as :

$$\pi_1 | T = c_1 - p_1 = \theta e^{-rT} P_0 .$$

Taking expectations over the time to death density $\phi(T)$ gives the first round option premium as $\pi_1 = \theta d P_0$, *where* $d = \int_0^\infty e^{-rT} \phi(T) dT$.

This is reinvested into a new perpetuity with coupon $y_1 = r\theta d P_0 = \theta d y$. The new perpetuity is used to write an additional call and buy a put, as before, and the process continued as described in appendix C for the straight call option. The resulting aggregate strike value and coupon are given by

(Aiv) $\quad X_b = \dfrac{(1-\theta)P_0}{1-\theta d}$

(Av) $\quad b(\theta) = \dfrac{y}{1-\theta d}$.

Comparability with the lifetime annuity swap can now be achieved by equating the total strike price X_b defined by expression *(Aiv)* with the desired terminal sum X_a under the swap. Equivalently, suppose a sacrifice ratio $\theta_a = 1 - \dfrac{X_a}{P_0}$ applies under the swap. Then the equivalent options sacrifice ratio is defined by

$$\theta = \frac{\theta_a}{1 - d(1-\theta_a)} .$$

It can be verified that with this choice the lifetime coupons generated by the put~call position using expression *(Av)* are exactly the same as generated under the swap under expression (1) of the text, just as one should expect.

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