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The zero capital approach to portfolio enhancement and overlay management

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Abstract

Both active and passive portfolio enhancement can be analysed within a zero capital framework, wherein enhancement exposures are reported as an additional or secondary portfolio requiring zero capital. This enables an identification of the economic value added by the enhancement, using two complementary approaches. The first is based on traditional beta analysis, which useful in identifying the direction and magnitude of exposures. The second is non parametric in nature and plots ordered mean difference schedules for the enhancement against the base portfolio. This enables risk profiling where the manager can match the likely range of his or her own risk preferences against the empirical history of the relationship, so that explicit risk premiums do not have to be utilized. The empirical illustration exhibits asymmetries in the effectiveness of currency overlay.

Key words: Benchmarking, currency overlay, equivalent margin, international diversification, ordered mean difference, portfolio enhancement, value at risk.

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I Introduction

Portfolio enhancement may be described as any activity that is designed to add value to a given reference portfolio. According to context, the latter may be taken as a benchmark, a base, a steady state, or simply an existing portfolio. A significant volume of internet commerce exists in connection with portfolio enhancement, but disproportionately little academic literature. One problem lies in defining just what is meant by enhancement, and how broadly the term should be defined. Fabozzi (1999) makes a distinction between active management, and enhancement. In his view, the former changes benchmark portfolio characteristics, but the latter does not. However, many people would consider portfolio insurance to be a form of enhancement, likewise the addition from time to time of higher yielding debt securities in the form of a credit spread play. Both of these activities could be expected to change the portfolio beta properties, and possibly other properties as well. Hallerbach (2001) identifies the enhancement contribution in terms of a marginal contribution to economic capital and interprets the latter as value at risk. Other authors have taken an even broader view of enhancement, to refer to any opportunities for improving the risk return tradeoff. For instance, Carow *at al* (2002) seek portfolio enhancement in terms of stocks with specific characteristics (size, book to market ratios etc) on the grounds of market undervaluation.

Implicit in the above are dual interpretations of the enhancement concept.

(a) *Strategic enhancement*, in the form of portfolio completion, as a more or less steady state welfare improvement. Here one would ask whether the addition of a new asset, or type of asset, has payoffs that can be expected to accomplish something which the existing portfolio cannot, in different states of the world. In other words, the existing portfolio does not span the payoffs now available with the addition of the new asset, and the spanning improvement can be expected to occur given the existing template for portfolio management style. Even a passive portfolio manager would appreciate this form of enhancement.

(b) *Active enhancement*. From time to time, the manager may wish to explore particular opportunities, which may be selective in nature, or may have their basis in market timing. Many financial institutions like to operate within this broad framework, which amounts to grafting an actively managed overlay on to a internal benchmark, or steady state, portfolio. Active enhancements may encompass portfolio insurance by means of derivatives, temporary portfolio reweightings, offshore investments and currency overlay management, and plays on credit spreads. The need to do so may be a matter of opportunity, changing risk attitudes as in portfolio insurance, or of temporary imperatives arising from market or institutional circumstances, as when trustee managers feel compelled to improve margins as a result of competitiveness in the market for their liabilities. Especially in the case of currency overlay, the active management is often farmed out to specialised managers.

The present paper takes a broad view of enhancement, as identifying any opportunities or imperatives that may cause the manager to depart from the base portfolio. Although we sometimes have in mind more or less temporary departures, the same methodology that identifies ex ante incremental welfare, relative to the base, should also be applicable to more permanent investment opportunities, resulting in a more or less permanent shift in the base portfolio. Thus it may not be too important to draw precise distinctions. Given a base portfolio, one asks whether is it possible to improve this, either by extending the selection domain or else by recognizing changed investor circumstances, such as changing investor risk premiums in contexts such as portfolio insurance. The same decision framework should be capable of handling both.

An second issue occurring in the literature is more operational in nature, namely how to measure the expected (or *ex ante*) contribution of any given portfolio activity to investor welfare. In deciding on any given enhancement activity, the manager has to have some sort of basis for judging whether it is likely to result in some improvement over the existing base portfolio. The first problem is how to isolate the incremental effect of the given activity. A second is how to represent the relationship of the activity to the base portfolio, and a third problem is how to assess its expected welfare improvement. The resolution of all three problems should ideally be undemanding in terms of their informational requirements. For instance, some institutions make an attempt to identify their institutional utility function for risk and reward, but this is clearly too stringent as a general rule and something much less demanding should be sought.

The incremental aspect is handled by encompassing the enhancement activity in its own portfolio, the enhancement portfolio, utilizes zero capital. At first sight, the zero capital aspect may seem a bit surprising. However, an SPI futures contract, or a credit enhancement swap, are examples of zero capital portfolios. They require no capital at inception, and indeed, our treatment throws some light on the practitioner's puzzle as to how to measure a return on a futures contract. Other forms of enhancement may utilize instruments that do require a capital input, e.g. portfolio reweightings, or options contracts, but these can be thought of as financed by going short the requisite dollar amount in the base portfolio. Specialised overlay managers commonly operate off a zero capital allocation, with limits on exposures. If so, their incremental returns are sometimes referred to as a 'portable alpha', with reference to Jensen's alpha as a performance metric.

The benchmarking and welfare problems can be handled in conjunction. Welfare increments can be assessed in term of an investor surplus concept called the equivalent margin in Bowden (1992, 2000), which can viewed as a risk adjusted return difference relative to the chosen base or benchmark return. The basic idea has linkages with welfare economics, and financial general equilibrium, as well as to fund performance measurement. An enhancement portfolio will add value if its equivalent margin is positive, bearing in mind that it uses zero capital. The equivalent margin performance metric improves over the 'portable alpha', which is subject to the well known Dybvig Ross (1985) critique of Jensen's alpha in the presence of market timing, which indeed is of the essence in active management. It can also be made less specific in its assumptions.

It then remains to make this criterion operational. Two ways are proposed to do this, both of them computationally undemanding. The first is based upon more or les traditional beta analysis, differing only in the zero capital aspect. An enhancement adds value if its expected return exceeds a beta adjusted risk premium. The beta formulation has the advantage of familiarity. In a zero capital framework, beta analysis can also help to make clear the dimensions of exposure to alternative proposed enhancements. However, it suffers from the parametric nature of the beta itself, together with the underlying mean variance assumptions in a portfolio context. For instance, returns data are rarely elliptical (e.g. normally distributed), and the welfare function can never be strictly quadratic.

A second methodology circumvents these difficulties by drawing on ordered mean difference methodology (Bowden 2000), which is non parametric in nature. The OMD enhancement schedule is a Lorenz curve type of construction, which plots the running mean of the engagement return against the benchmark returns. By examining the slope and sign of the schedule, one can see whether the proposed enhancement is suited to the risk profile of the manager. The OMD schedule is related to the absolute concentration curve of Shalit and Yitzhaki (1994) used to test for stochastic dominance; the normalisation involved converts this to the return metric required by the equivalent margin welfare measure. Cognate references are Post (2001), Bowden (2003) in a SD efficiency context, Markowitz (1959), Russell and Yeo (1988), and Bowden (2000, 2002), for the basis in utility theory in the form of utility generators. In the present context, however, stochastic dominance will not usually be present or even aimed for. The objective is oriented towards performance measurement on a bilateral basis against a designated benchmark, with the aim of deriving expected incremental return numbers as a diagnostic to assistport folio reallocations.

The scheme of the paper is as follows. Section II establishes the zero capital enhancement framework, drawing on a number of common applications. Section III reviews the investor surplus concepts to be used, establishing the enhancement version of the equivalent margin. The beta based implementation is described, and it is shown how this might be utilized to determine intrinsic exposures. Section IV develops the non parametric version. The illustration is empirical and shows how to assess the benefits or otherwise of currency overlay management. It is noted that these are not necessarily symmetric, as between foreign and local residents, and there are some substantial differences as between different currency pairs. Section V contains observations on active management and concluding remarks.

II The portfolio enhancement margin

A basic issue in enhancement is to determine the incremental value of a portfolio action, the latter viewed as some departure from a benchmark or base portfolio. One way to do this is to isolate the active element as a portfolio in its own right, but one that requires no dedicated capital. Positive returns arising from this supplementary portfolio are therefore pure gains relative to the base portfolio. Thus the enhancement framework to be used can be regarded as adding a second portfolio (the enhancement portfolio, indexed by 'a') to that of primary interest (the base, or benchmark portfolio, indexed by the letter 'b'). In pure enhancement, the additional portfolio carries no capital, meaning that its upfront value is zero. Returns on the enhancement portfolio are then expressed relative to some arbitrary face value, in such a way that that the sum of portfolio weights in the enhancement portfolio will be zero.

More formally, let V_{b0} be the initial value (investment capital) to the base portfolio, and let r_b be the return on the base portfolio. The enhancement portfolio can be thought of as comprising a set of assets of nominal or face value A_{0i} and returns r_{ai} relative to that face value. Then the total return on the enhanced portfolio is given by

$$(1) \qquad r_T = r_b + r_a \quad where$$

$$r_a = \sum_i a_i r_{ai} \ ; \quad a_i = \frac{A_{0i}}{V_{b0}} \ with \sum_i a_i = 0.$$

Thus the weights of the enhancement portfolio sum to zero, rather than the usual unity, and reflect the incremental nature of the enhancement return. Below are some examples. Some enhancement portfolios naturally entail zero capital. Futures or swaps provide an example, for derivatives of this kind require no capital at inception. Others have to be constructed to have this feature: if the proposed additional investments require capital, then of necessity this is at the expense of the base portfolio, so that there are short elements of the latter in order for the enhancement to be self financing. The examples that follow illustrate both types.

Example 1 A futures contract in enhancement terms

Suppose the manager adds to the existing base portfolio of stocks a long SPI futures contract with face value $$F$. This can be regarded as two investments:

(a) Long *$F* worth of an asset with end of period accumulated value $(1+r_{a1})F$. Here the return r_{a1} is that on the notional face value, so if the face value is $1,000 and the close out value of the contract is $1,100 then r_{a1} = 10%.

(b) Short an asset of the same face value, namely $$F$, but with a notional return of

$r_{a2} = 0\%$.

So the end of period value of the enhancement portfolio is

$$A_1 = F(1+r_{a1}) - F(1+r_{a2}) = F\,r_{a1} - F\,r_{a2} \quad .$$

Let V_{b0} be the initial value of the base portfolio, and let r_b = the return on this portfolio. The end of period value to the base portfolio is $V_{b0}\,(1{+}r_b\,)$. The combined or total end of period value V_T and return r_T are therefore given by

$$V_T = V_{b0}(1+r_T) = V_{b0}(1+r_b) + Fr_{a1} - Fr_{a2} \quad .$$

Divide both sides of this equation by the initial capital V_{b0} . Also define 'weights' for the enhancement element as

$$a_1 = \frac{F}{V_{b0}} \quad and \quad a_2 = \frac{(-F)}{V_{b0}} \quad .$$

Notice that $a_1 + a_2 = 0$, so that the weights in the enhancement sum to precisely zero - the enhancement portfolio has a capital application of zero, as required. We can now write the total return in the required form (1) as

$$r_T = r_b + r_a \quad ,$$

$$where \quad r_a = a_1 r_{a1} + a_2 r_{a2} \text{ ; } and \ a_1 + a_2 = 0 \quad .$$

Remark: Measuring the return on a futures position is an issue that has troubled practitioners from time to time. Here it does not arise. It is always simply the return on the face value of the contracts involved. The indeterminacy is transferred to the weights in the enhancement portfolio, for now a_1 and a_2 will depend upon the value of the base portfolio.

Example 2 Portfolio reweighting as an enhancement

A reweighting of the portfolio, perhaps to favour more defensive assets in riskier times, can be represented as a zero capital enhancement[1]. The longer position in the

[1] For instance, suppose the base portfolio has three assets with weightings w_1 , w_2 , w_3 summing to one. We wish to change the first two to w_1', w_2', keeping the third weight unchanged, but weights all still summing to unity. This can be accomplished by an enhancement portfolio by setting weights $a_1 = w_1' - w_1, a_2 = w_2' - w_2, a_3 = 0; \ so \ a_1 + a_2 + a_3 = 0$ with notional capital $A_{10} = (w'_1 - w_1)V_{B0}, A_{20} = (w'_2 - w_2)V_{B0}, A_{30} = 0; \quad A_0 = A_{10} + A_{20} + A_{30} = 0.$ The revised portfolio return is then the sum of the base return and the enhancement portfolio return, the latter having weights $\{a_i\}$ that sum to zero.

defensive asset is effectively financed by going short against the existing riskier assets. The simplest case is a *cash enhancement*. Suppose the manager wishes to protect the base portfolio by going long in cash of amount \$C, with certain return ρ, financed by going short in the base portfolio. The notional capital committed is zero and the return on the enhancement portfolio is

$$r_a = \frac{C}{V_{b0}}(\rho - r_b) \quad .$$

This falls into framework (1) above.

Example 3 Options as an enhancement

Unlike futures, options commit capital, in the form of the purchase premium. However, the latter cash component can be handled just as for cash enhancement, by debiting the base portfolio at an opportunity cost of r_b .Suppose the manager goes long 1 call option on the base portfolio, financing the option premium π by going short to that value in the base portfolio. This is a zero capital portfolio. Its return may be defined as follows. Let S_0 be the current price of the stock to which the option unit carries entitlement, and S_1 is the end of period price. Then the option's return r_c is defined by

$$1 + r_c = \frac{(S_1 - S_0)_+}{\pi} = \frac{S_0}{\pi} r_b^{\,+} \quad ; r_b^+ = \max(r_b, 0) \,.$$

The end of period value of the entire portfolio is given by

$$V_1 = V_{b0}(1 + r_b) + (1 + r_c)\pi - (1 + r_b)\pi \ ,$$

Defining zero sum weights $a_1 = \dfrac{\pi}{V_{b0}}; a_2 = -\dfrac{\pi}{V_{b0}}$, we have $r_a = a_1 r_c + a_2 r_b$ and

$r_T = r_b + r_a$.

Example 4 Foreign exchange (currency) overlay as enhancement

Let r_b stand for the return on a portfolio of domestic assets, taken as the base portfolio. The manager is considering adding a foreign portfolio of intrinsic or hedged return r^*. The latter may supplemented by a possible exchange rate return denoted by e, the percentage depreciation of the home currency against that in which the foreign return is denominated. Alternatively the FX exposure may be hedged by means of a costless procedure such as forwards or currency swaps.

Let a proportion g of the foreign exchange exposure be hedged and a proportion $(1\text{-}g)$ remain unhedged. Suppose the foreign transactions are to be financed by means of $\$C$ diverted from the base portfolio, amounting to going short C against the full amount . Thus the value created with the foreign component is equal to $C[gr^* + (1\text{-}g)(r^* + e)] = C[r^* + (1\text{-}g)e]$. The enhancement return is given by

(2) $$r_a = \frac{C}{V_{b0}}[r^* - r_b + (1-g)e].$$

Two sources of value arise. The first is the foreign asset portfolio enhancement arising from the difference between domestic and overseas investments. The second is the currency effect, to the extent that the exposure is unhedged. One could call it the currency overlay.

It is possible to list further examples. For instance, a standard insurance contract on property is an enhancement portfolio. Its definition parallels that of the option above. At this point, however, it will be useful to look more closely at the underlying notion of a return that utilises zero capital, and what this is to mean.

Enhancement and scale

Recalling expression (1) above, the enhancement return r_a is defined implicitly by the relationship

(1)' $$r_a V_{b0} = \sum_i r_{ai} A_i \,.$$

The right hand side is the dollar earnings from the enhancement portfolio and the left hand side expresses this as an equivalent return on the given base capital V_{b0} . Defined in this way, the enhancement return r_a is not independent of the scale of the enhancement process; doubling all the enhancement exposures A_i will double the measured return r_a .

Because of the zero sum property, one can define the unit level of an enhancement portfolio at will. In most cases this would refer to a natural unit such as a single futures or options contract, or an extra dollar of cash enhancement. Thus if r_a is the normalized or unit enhancement return, a portfolio with return $\tilde{r}_a = xr_a ; x > 0$, is also an enhancement in the same direction. In the framework that follows, we shall look at the consequences of variations in the enhancement exposure x, with the unit level taken as understood. An enhancement will be value creating if increasing the exposure x, relative to some base level, increases the measured return $r_b + xr_a$ on the fund.

III The equivalent margin and incremental value

Is enhancement worthwhile, and in what circumstances? It will be worthwhile if it increases the expected utility of the investor. In the case of managed funds, we will maintain the hypothesis that managerial utility functions are correctly aligned with those of their clientele investors. Subjective value is assumed to be measurable in terms of expected Von Neumann Morgenstern utility $E[U(r_T)]$, where U is some real valued concave (risk neutral or risk averse) utility function defined on portfolio returns r_T . Initial wealth and other modifiers are assumed to be taken as given. In other words, the full utility function is of the form $U(W,W_0)$, with $W = W_0(1+r_T)$, but we suppress the initial wealth in what follows.

The precise nature of the utility function will not concern us at this stage. In some cases, this emerges naturally from the nature of portfolio constraints; value at risk (VaR) methodology is a case in point (considered below). Apart from this, however, a striking feature of the OMD approach is that much can be said without having to specify the precise functional form of the underlying utility function. For instance, one can identify sources of welfare gain to portfolio readjustments in response to changes in the utility function, without knowing too much about just what the function itself might be.

Welfare measurement will be approached through the window of the equivalent margin, also used in ordered mean difference analysis. The equivalent margin can be called the investor surplus generated by the instrument or portfolio r_a relative to the benchmark. Consider the following notional experiment. Assume that an enhancement portfolio is available and we wish to assess whether this will add utility value. Imagine that we can tax this portfolio return at a flat rate tax t per unit held, so the net return is
$r_a - t$. The equivalent margin is obtained as the value of t that finally drives the holding or scale x of the enhancement return to zero. It can be either positive or negative. Readers familiar with welfare economics will recognize this idea as the equivalent variation. The optimized deprival value in capital budgeting and monopoly pricing is a similar concept.

For a fixed t , the enhancement portfolio problem becomes

$$\max_x E[U(r_b + x(r_a - t)] \,,$$

and the solution $x = x(t)$ satisfies

(3) $$E[(r_a - t)U'(r_b + x(t)(r_a - t)] = 0 \quad .$$

The function $x(t)$ is monotonically declining with t, which amounts to a sure tax, and is therefore progressively more unpleasant. The equivalent margin, or optimized deprival value, is the tax t_0 such that $x(t_0) = 0$, and is obtained from (3) as:

(4) $$t_0 = \frac{E[r_a U'(r_b)]}{E[U'(r_b)]} \quad .$$

Thus the equivalent margin is the benchmark utility weighted expectation. A positive margin t_0 means that adding units of the enhancement portfolio will result in a combined portfolio that is expected utility improving over the base or benchmark portfolio. Recall that the enhancement portfolio utilises no initial capital. All that is required is that its risk weighted return be greater than zero, and the benchmark return provides the appropriate risk weights as $\pi(r_b) = E[U'r_b)]/E[U'(r_b)],$ interpreted as state price deflators in the language of financial general equilibrium theory (e.g. Duffie 1992). However, there is no necessary reference to market equilibrium in the present context, which is one of portfolio theory at the individual level.

The equivalent risk free rate (ERFR)

Given that the equivalent margin for the enhancement portfolio consists in a risk weighted expectation of return, it is useful to have available a similar benchmark but applied to the base portfolio itself. The quantity designated

(5) $$\rho_b = \frac{E[r_b U'(r_b)]}{E[U'(r_b)]}$$

will be referred to as the equivalent risk free rate (ERFR) for the benchmark portfolio. To see why, imagine cash enhancement as in example 2 above, that carries an equivalent margin $t_0 = 0$. In other words if $r_a = \rho - r_b$, then there is no incentive to go either long or short the benchmark. The ERFR solves this equation to give $\rho = \rho_b$. It will be greater for less risk averse investors. The ERFR does not rely on the existence of an actual risk free rate, but if one does exist, then the two should be equal if the base portfolio is to be in portfolio equilibrium with the given risk free rate. Otherwise there will be an excess return to a cash or futures enhancement[2] proportional to the difference $\rho - \rho_b$.

Let μ_b be the mean of the base portfolio. The magnitude

$$GR_b = -\frac{E[(r_b - \mu_b)U'(r_b)]}{E[U'(r_b)]} \quad .$$

may be called the generalized Rubinstein risk premium, because it reduces[3] to the Rubinstein (1973) version of the risk premium, namely $-\frac{1}{2}\sigma_{r_b}^2 E[U''(r_b)]/E[U'(r_b)]$, in the special case where base returns r_b are normally distributed (not assumed here). We can write

(6) $\quad \mu_b - \rho_b = GR_b \quad .$

The difference between the expected value of the base return and the equivalent risk free rate can be taken as a risk premium, one intrinsic to the particular investor with utility function U. In market equilibrium, the risk premium is generated by the GR risk premium of a representative investor, so that this becomes a more suitable form of the CAPM risk premium whenever returns are not normally distributed. Bowden (2002, Appendix) contains a diagrammatic interpretation of the GR premium applied to a personal portfolio optimum; it is numerically greater than the corresponding Pratt Arrow risk premium.

A proposed enhancement with zero capital return r_a will be said to be *ex ante effective* if $t_0 > 0$. Given that zero capital is involved, the only hurdle that the proposed

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[2] The cash enhancement difference follows directly from the discussion of section II. In addition, suppose we price an SPI futures contract as a forward, and imagine for simplicity that the underlying SPI pays no dividends over the period. Thus the current value of the contract is $F_0 = S_0(1+\rho)$ and the end of period value will be $F_1 = S_1$. As in example 1, the return on the enhancement portfolio will be

$$r_a = \frac{F}{V_{b0}} \frac{r_b - \rho}{1+\rho} \quad .$$

It can be verified by using expression (4) that apart from the scale factor F/V_{b0} , the equivalent margin is given by

$$t_0 = \frac{1}{1+\rho}(\rho_b - \rho) .$$

This is a long futures enhancement. A short futures based enhancement, as in portfolio insurance, is effectively the same as a cash enhancement.

[3] This follows from Price's lemma, one version of which (Bowden 1997) states: If x,y are Gaussian, then

$$E[g(x)h(y)] = E[g(x)]E[h(y)] + \frac{1}{2!}\sigma_{xy}E[g'(x)]E[h'(y)] + \frac{1}{3!}\sigma_{xy}^2 E[g''(x)]E[h''(y)] + \ldots$$

For special cases, see Stein (1973) and Amemiya (1982).

enhancement return has to satisfy is that its risk adjusted expectation exceeds zero. As is stands, however, such a requirement is not fully operational, and the discussion that follows addresses this issue in two ways. The first is a more conventional beta framework, and the second is the non parametric OMD framework.

The beta approach to enhancement

Write the conditional expectation $E[r_a|r_b]$ in the form

(7) $$E[r_a|r_b] = \mu_a + \beta_a(r_b)(r_b - \mu_b) \quad ,$$

where $\mu_a = E[r_a]$, $\mu_b = E[r_b]$, and $\beta_a(r_b)$ is an unspecified function of r_b . The leading case is where the conditional expectation function is linear in r_b, in which case $\beta_a(r_b) = \beta_a$, constant .

Using the iterated expectation, and expression (7), the equivalent margin (4) becomes

(8) $$t_0 = \mu_a - \beta_a(\mu_b - \rho_b) \, .$$

Using (8) together with (6), the enhancement will be worthwhile if

(9) $$\mu_a > \beta_a GR_b \quad .$$

The breakeven condition is that the conditional mean of the proposed enhancement instrument of portfolio must exceed the base risk premium adjusted by the enhancement beta measured against the base portfolio. The higher the beta, the greater is the hurdle for enhancement to be worthwhile. Note the generic similarity to the conventional CAPM, with r_b playing the role of the market portfolio. The difference is that because the enhancement portfolio involves no net capital, the risk free rate is missing.

In active enhancement, the risk premium denoted GR_b may vary, giving rise to cyclical variations in the desire for enhancement. In classic portfolio insurance, as the market rises, and client wealth with it, the risk premium will diminish, perhaps making it easier for the manager to meet the enhancement hurdle (9). This might take two forms.

(a) *Aggressive enhancement*. Here the manager wants 'more of the same' as the benchmark, seeking to enhance along the same direction of risk exposure. The manager will choose an enhancement that has positive beta on the benchmark.

This is likely to preferred in a bullish market, where the risk premium is diminishing.

(b) *Defensive enhancement*. This is designed to protect against losses in the base portfolio, and takes advantage of a negative beta. Condition (9) continues to apply. However with a negative enhancement portfolio beta, the enhancement instrument can add value even where its own expected return is zero or negative. An enhancement that would not survive as an investment in its own right acquires value as a defensive hedge. A rising risk premium as the market declines will make it easier to adopt a defensive enhancement.

In the beta approach, expression (9) helps to formalize the different dimension of the enhancement decision. Different values of β_a and μ_a characterize different possible enhancements. The choice as to which will yield most value is also informed by a possibly changing risk premium, so that decisions are dynamic and depend upon the state of the world at the time. For a cash based enhancement as in example 2 section II,
$\beta_a = -1$, and for the corresponding short futures position it is $\beta_a = -1/(1+\rho)$.

Credit based enhancement is an interesting case. Fund managers, especially those whose business is based on achieving margin over liabilities, often resort to this to generate extra margin. It could be done in several ways, not all of which are equivalent in terms of the enhancement betas:

(a) Via a portfolio reweighting, going longer in lower grade credits and short in the base portfolio. Lower grade credits have a beta which is typically greater than unity. In good times, credits spreads narrow and in bad times they widen. Thus if β_c is the inherent beta of the credit spread, the net enhancement beta against the base portfolio is $\beta_a = \beta_c - 1$.

(b) Via credit derivatives such as credit swaps. In the case of a swap there is no upfront commitment of capital, and the enhancement beta is effectively just $\beta_a = \beta_c$.

Thus credit derivatives are a particularly levered form of enhancement with corresponding break even demands on the expected return μ_a .

A problem with even the beta based enhancement is that one may not have a firm idea of the institutions' risk premium; nor is the beta relationship necessarily linear. The

OMD non parametric enhancement helps to solve this problem, and will be considered in the next section. We end the present section with an example where the beta approach can be combined with additional information about risk attitudes to derive a fully operational ex ante enhancement test.

Example: VaR based enhancement

Value at risk (e.g. RiskMetrics 1996, Wilson 1995) can be regarded as implying a dichotomous managerial utility function with heavy penalties if the return falls below a designated floor r_L , implying a loss of capital below a certain tolerance. One could represent the floor in the form $r_L = \mu_b - \sigma_b \alpha$, where σ_b is the standard deviation of the base return and α is a tolerance parameter

The simplest such utility function would be linear above this floor and impose a flat penalty *X* if returns fall below the floor. Letting $X \to \infty$ implies a prohibition on returns falling below the floor. Thus if R is the portfolio return, the implied utility function can be written as $U(R) = R$ *if* $R \geq r_L$; $U(R) = -X$ otherwise.

The equivalent risk free rate is given by

$$\rho_b = r_L + \frac{\int_{r_L}^{\infty} (r_b - r_L) f(r_b) dr_b}{1 - F(r_L) + (r_L + X) f(r_L)} \quad ,$$

where $f(r_b)$ and $F(r_b)$ are the density and distribution function of the base return. As the penalty *X* rises, the equivalent risk free rate drops. Letting $X \to \infty$, the ERFR becomes equal to r_L .

If the penalty $X \to \infty$ the equivalent margin is given by

$$t_0 = \mu_a - \beta_a \sigma_b \alpha$$

and the enhancement condition becomes

$$\mu_a > \beta_a \sigma_b \alpha \ .$$

Notice that a more stringent VAR limit, which has a smaller value of α, imposes less stringent conditions on the expected return of the enhancement portfolio. This is the clutching at straws syndrome: The closer to the lower boundary, the higher the effective returns to any degree of enhancement.

IV A non parametric approach to enhancement

Ordered mean difference enhancement theory does not assume that the underlying regression relationships are linear. Moreover, it enables a differential assessment according to the perceived risk band of the user. Even if the user does not know his or her precise position along the resulting one dimensional risk spectrum, there may be an appreciation that the position has changed and this can be exploited.

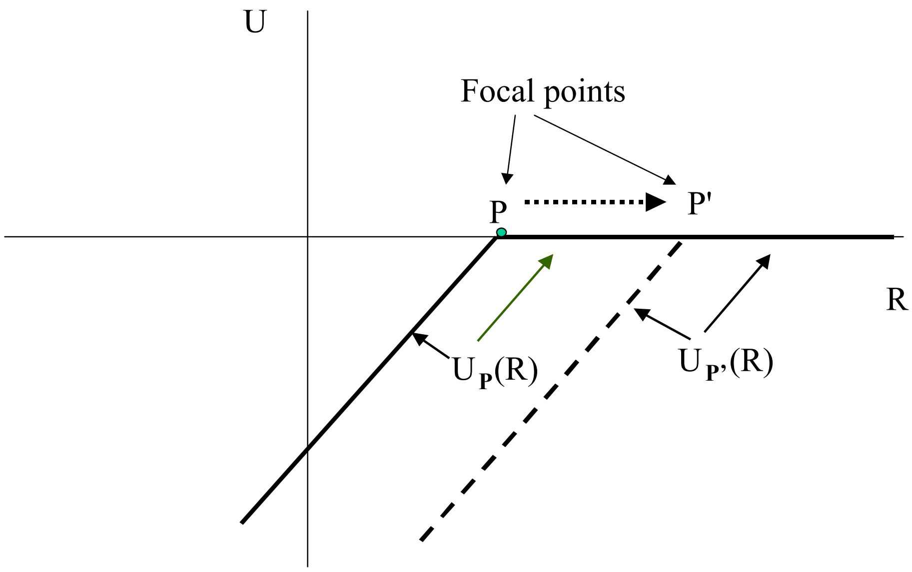
The starting point is to assume a particular form of the utility functions U, portrayed in figure 1 below as a piecewise linear function with a focal point or node P at which point it changes slope from unity to zero. One can write such a function in different ways, for example as $U_P(R) = \min(R - P, 0)$, where the parameter P has a return dimension, i.e. is commensurate with R. In analytical work it is more convenient to express such functions in terms of generalised functions, which allow differentiation even at points of discontinuity[4].

[4] The OMD utility generator is technically defined by $U_P(R) = (R - P)SF(P - R)$, where the step function SF(.) is defined by

$$SF(x) = \begin{cases} 0\,; x < 0 \\ \frac{1}{2}; x = 0 \\ 1\,; x > 0\,. \end{cases}$$

For further properties and the underlying mathematics, see Lighthill (1959), Antosik *at el* (1973), or Vilenkin and Klimyk (1995).

Figure 1 Generator utility function $U_P(R)$



The function $U_P(R)$ is called an OMD utility generator, for it can be shown (Bowden 2003), that the expectation of any risk averse utility function can be decomposed into a weighted sum of the expected generators. The expected value at any point P of the OMD generator also gives the area underneath the cumulative distribution function (Russell and Yeo 1988), so they could alternatively be called the SD generators, as this area forms the basis for second order stochastic dominance and distributional majorisation theory. Sketched in figure 1 are two generator utility functions with nodes at the points P, P' . As the nodal point P increases, the associated generator utility function become progressively less risk averse.

For the utility generator at P, the enhancement margin is given by

$$t_0(P) = \frac{E[r_a U_P'(r_b)]}{E[U_P'(r_b)]}$$

The marginal utility $U_P'(r_b)$ has value unity for $r_b < P$, and zero for $r_b > P$, and we set the derivative at unity for $r_b = P$. Multiplying by the marginal utility therefore amounts to a censoring device, preserving only those values for which $r_b \le P$. With discrete data, this amounts to computing a schedule

$$\frac{1}{\#\{r_b \le P\}} \sum_{r_b \le P} r_a \ ,$$

in which the observation pairs (r_a, r_b) are first ordered by increasing values of the benchmark r_b . One then takes the mean of r_a up to the given point P. We shall call such an empirical schedule the OMD enhancement schedule.

Definition: *Given a time series of observation pairs* $(r_a,\ r_b)$*, for a return or return element* r_a *against a benchmark* r_b *, the OMD enhancement schedule plots the running or censored mean of* r_a *over* $r_b \le P$ *against* P .

In practice, this operation is easily accomplished with an Excel or Lotus spreadsheet, using the *Data/sort* menu facility in Excel to reorder the observations by ascending values of the base r_b . The enhancement schedule renormalises the OMD schedule in Bowden (2000), which is based on return differences, to the zero capital context. The OMD schedule can itself be regarded as a metrification to an equivalent return dimension[5] of the absolute concentration curves used by Shalit and Yitzhaki (1994) as a test for stochastic dominance. The equivalent return can then be used in a fund performance context and acquires interpretive value as an equivalent margin or investor surplus even where stochastic dominance cannot be expected[6].

Risk profiling

As indicated above, the name utility generator is given because the space of utility functions is effectively spanned by the utility generators. In the present context, this can be shown to imply that for an arbitrary risk averse utility function U, the associated enhancement margin can be written in the form

[5] The sample versions of the Shalit and Yitzhaki construction defines the absolute concentration curve (ACC) for security j as the progressive cumulated return as censored by the return on an assumed base portfolio. Take the base return of another such security, subtract from the ACC for security j, and divide at each base return ordinate by the sample number to that point to create a running mean. The result is the OMD of security j against the benchmark. The enhancement version would just use the mean version of the ACC for security j.

[6] An alternative approach might be to construct stochastic dominance efficient portfolios utilising combinations of the enhancement and base. Investor welfare improvement would then be based on a comparison of the base portfolio with an efficient portfolio. But since there is a range of the latter, in the form of an efficient frontier, we are back at the utility dependence problem. However, there is a duality or gradient relationship under which the OMD is related to optimal stochastic dominance portfolios (Post 2001, Bowden 2003), suggesting that use of the OMD in welfare improvement will achieve much the same results as based on the full SSD efficient frontier.

$$t_0 = \int_{-\infty}^{\infty} w(P) t_0(P) dP$$

where the weights w(P) are semipositive and sum to unity (Bowden 2001). They are explicitly given by

$$w(P) = \frac{(-U''(P))F(P)}{E[U'(r_b)]};$$

so that the weights depend upon the profile of second order derivatives for the given utility function. More risk averse people have a weight function that is more concentrated on lower values of *P*.

The spanning property also implies that in his or her optimal portfolio decisions, any risk averse agent will act as though he or she operates off a single utility generator utility function with a node P_* that depends upon the underlying utility function. This is the 'representative gnome' result. If such an agent becomes less risk averse, then P_* will shift to the right. The most common range of risk aversions corresponds to *P* values in the vicinity of whatever is the current risk free rate in the economy.

Thus if the OMD enhancement schedule $t_0\ (P) > 0$, for every *P*, we can assert that the enhancement will add value no matter what the utility function actually is. Even if this is not the case, the profile $t_0(P)$ gives us a clue as to how people of differing risk propensities would view the enhancement. If the schedule $t_0(P)$ is positive for lower values of *P* , but not for higher, this will mean that the enhancement is adding value for the more risk averse, but not necessarily for the less risk averse. The following example illustrates the above methodology.

International enhancement

Suppose that the manager of a domestic portfolio is considering an international enhancement as in example 4 of section II. Using expressions (2), (4) and the utility generator, we obtain

$$t_0(P) = \frac{C}{V_{b0}} \{ \frac{E[(r^* - r_b)U_P'(r_b)]}{E[U_P'(r_b)]} + (1-g)\frac{E[(eU_P'(r_b)]}{E[U_P'(r_b)]} \},$$

where it will be recalled that *1-g* is the proportion of the base remaining unhedged.

The total OMD enhancement graph can therefore be decomposed into the two parts, the first due to the pure foreign index return, the second to the unhedged currency component.

(a) The first component is the classic OMD schedule of the foreign return r^* against the home portfolio return r_b taken as benchmark.

(b) The second amounts to a moving average of the exchange rate changes once the data are ordered by increasing values of the benchmark

One can examine the two components separately. The first issue is whether the foreign investment will enhance the domestic portfolio and the second is whether the foreign exchange component adds value in its own right.

Figures 2 and 3 plot OMD enhancement schedules for the two components from two different stances, using monthly return statistics over the period August 1982-August 2002. Data source is Thomson Financial Datastream.

(a) Figure 2 assumes a U.S. domestic portfolio comprising the MSCI U.S. index and an investment in the corresponding U.K. MSCI index. The exchange rate is the GBP/USD WMR midrate, so if this rises then the U.S. investor wins.

(b) Figure 3 assumes the opposite stance, looking at the gains to a U.K. investor from investing in the U/S. index.

All OMD enhancement schedules (bold) have approximate one sigma (standard error) confidence bands attached (light hatched). For the construction of these, see Bowden (2000). Some near symmetries can be expected in the resulting enhancement schedules. Thus the hedged return differences will be common to investors in both countries, so that the OMD schedule for the one will tend to reflect that of the other about the horizontal axis. The only difference will be the adjustment by differing base portfolios, so that the reflection will not be perfect. Likewise, movements in exchange rates are often associated with movements in exchange rates, so one blips in the one might sometimes be reflected in the other, though the association will be weaker than for the hedged return differences.

The most striking feature of the figures is that they reveal different benefit profiles depending upon who invests where, even between a given country pairing.

(a) From figure 2, the enhancement story is mixed for U.S. domiciled investors. The hedged return component is never large. It is unfavourable for very risk averse investors, while the currency component is favourable.

(b) Figure 3 shows that U.K. index investors would unequivocally benefit from investing in the U.S. Index. But because the currency component is negative, they would also be unequivocally better off by complete hedging of the foreign exchange exposure.

Figure 2 Enhancement for US investor with UK index

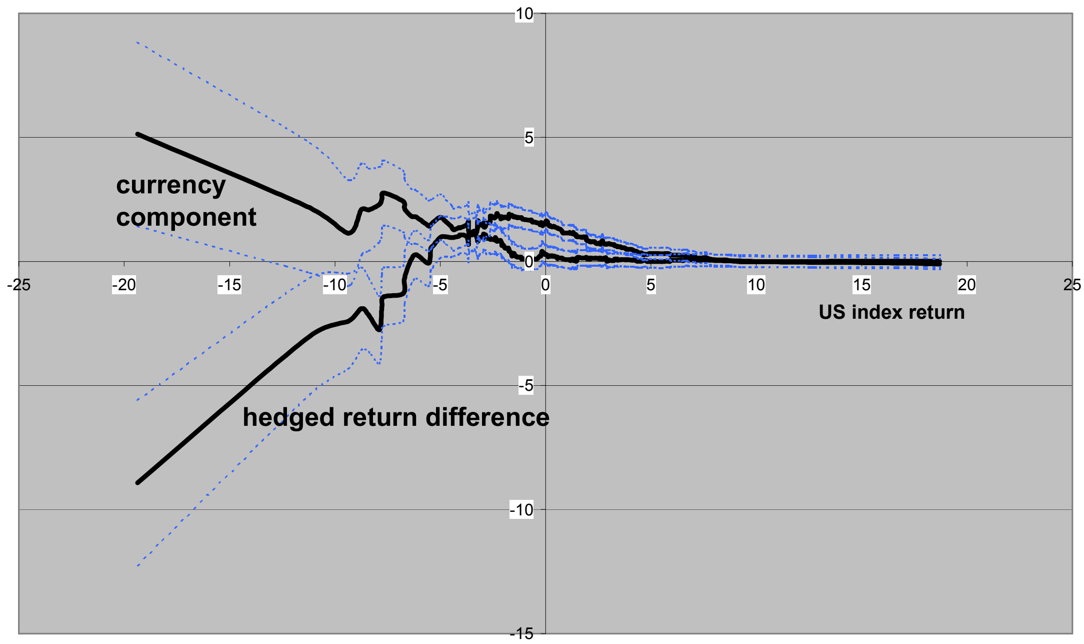


Figure 3 Enhancement for UK investor with US index

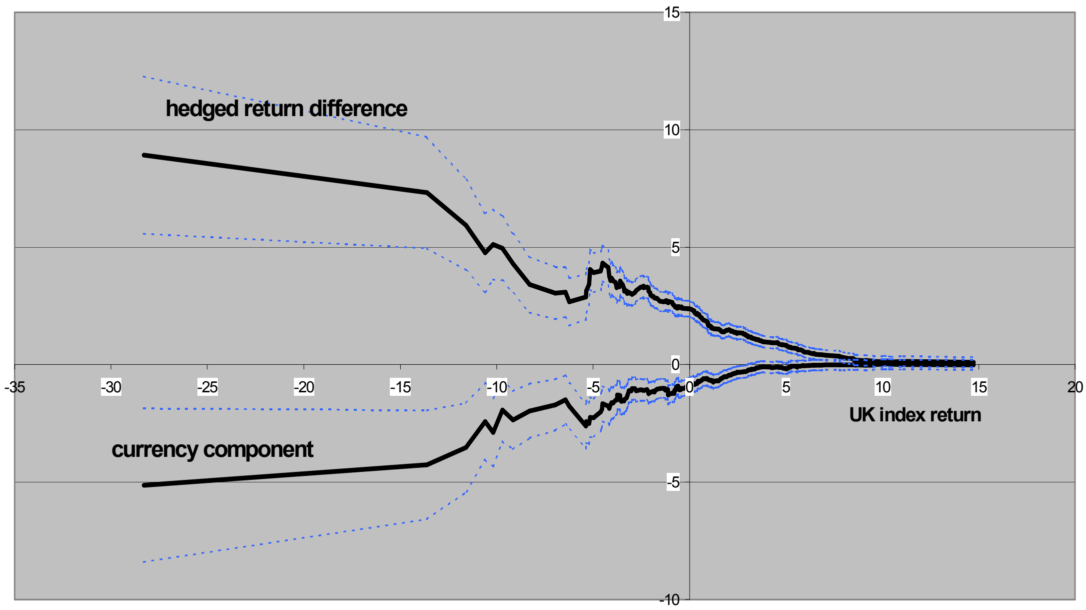
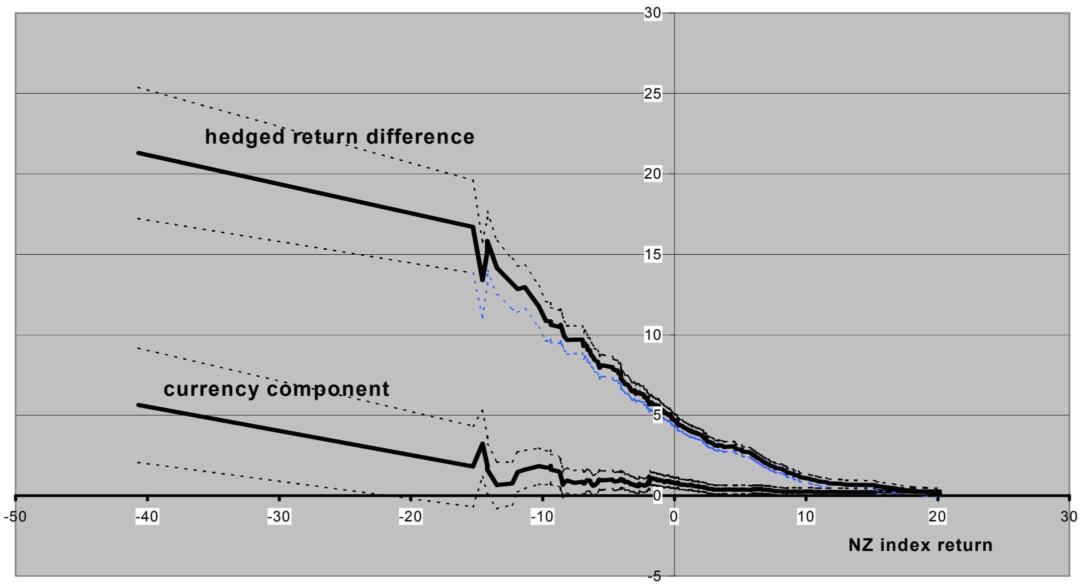


Figure 4 below shows an example where both the hedged and currency components are favourable. A New Zealand investor enhancing via the US index would benefit from both components.

Figure 4 Enhancement for NZ investor with US index



OMD schedules versus the beta approach

The OMD schedule has two advantages over the beta approach: First, one does not need to assume that the underlying theoretical regression (i.e. conditional expectation) of r_a on the benchmark r_b is necessarily linear. Secondly, one need not know very much about the risk characterization of the user. As the above example showed, it nevertheless remains possible to infer useful guides to action.

Some common ground exists between the two. Both can be regarded as instrumentalising the equivalent margin, or investor surplus, welfare measure. They both rely on some underlying regression relationship, even if this is not precisely known. In the case of shifting risk preferences, one would have to assume that this regression relationship remains stable, i.e. is not itself affected by the altered attitudes to risk on the part of the market as a whole. In other words, there is an implied decomposition according to which the underlying regression, whatever it might be, stays the same but shifting risk attitudes imply movements in the representative nodal point P_* to and fro along the horizontal axis. Thus the representative point P_* becomes an index of risk preferences.

V Further remarks and conclusions

Active and passive enhancement

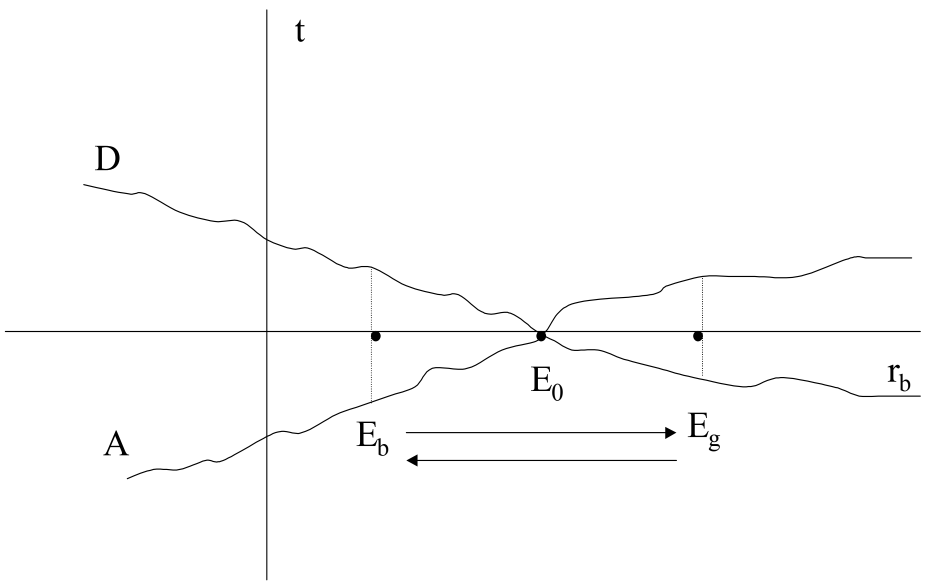
Both approaches – beta and the OMD- can be used in active as well as passive contexts. If one is to base enhancement decisions on historical data, the applicability test for either technique is whether observed regression relationships can be assumed invariant over time. In portfolio insurance and hedging contexts, such an assumption is implicit. As the market moves, the manager reacts to changing wealth, in the form of the value of his or her portfolio, and portfolio insurance represents a response to changing market or portfolio risk premiums. The manager may seek refuge in assets known to be defensive in character, on the basis of their historical betas, or in of the slope of the OMD schedule, against the benchmark. He is betting that the bilateral relationship will stay the same even though the prospects for the benchmark return may have changed.

Given the stability of presumed regression relationships, one can see how enhancement can occur in portfolio insurance, under either the beta or the OMD framework. For the purposes of illustration we assume that the manager has available two alternative enhancements, both with the same expected return, but with quite different covariance profiles with respect to the benchmark.

(a) *The beta test*. In this decision framework, a proposed enhancement will occur if $\mu_a > \beta_a GR_b$. During a market upswing, the risk premium GR_b will decline with increasing wealth, triggering the above condition even for positive beta assets. Thus an asset that might not have been chosen hitherto now becomes seen as value enhancing. During the downturn, the risk premium will rise, and such positions will be unwound.

(b) *The OMD test*. Figure 5 sketches OMD schedules for two proposed enhancements, one aggressive in nature (positive slope) and the other defensive (negative slope). Suppose that initially both are in equilibrium with the benchmark portfolio, so that in each case their OMD value is zero, at the point E_0.

Figure 5 OMD enhancement with changing risk premiums



As the market rises, and times are good, managers will become less risk averse. This means that the representative utility generator for any such manager will shift to the right, occupying a higher value of the node P (c.f. figure 1 of section II). This is indicated at the new personal equilibrium point E_g in figure 5. The OMD value for asset A is now positive while that for the defensive asset D is negative. Hence the manager will favour aggressive enhancements and download defensive ones. The process goes into reverse in the market downswing and times are **b**ad. Now the representative utility generator has shift to the left, marked in as E_b, and the manager will favour defensive enhancements and download the more aggressive ones.

The beta approach does not specify that the manager actively unload defensive assets as times become good, for such assets will continue to satisfy the beta condition. The OMD approach tells us a bit more: the manager will actively down weight the defensive assets in good times. As earlier remarked, the OMD schedule approach is less demanding in other respects, e.g. it does not assume that the regression relationships are necessarily linear.

Marginal versus total exposures

The beta or OMD approaches are both marginal in nature. They are conditions under which adding a small amount of a given enhancement will increase investor welfare; by themselves they do not address the issue of just how much can safely be added. Recall from section II that a zero capital enhancement remains a zero capital portfolio when scaled up by any arbitrary amount. The effect of any such scaling will be to increase the exposure potentially without limit, so the issue of just how much is safe has to be addressed.

More formally, let r_a be the return on a unit level of a given enhancement, and consider a total portfolio return of $r_b + xr_a$ .If the investor utility function were known one could solve for the optimal scale of enhancement in terms of a formal expected utility maximization process. Less formal treatments might specify a VAR type approach. During the upswing, wealth has increased and investor risk aversion has decreased. Given that a proposed enhancement has been revealed (beta or OMD approaches) to add value, one would set the level x to satisfy prespecified VAR limits, where the latter would also reflect the diminished risk aversion.

Concluding remarks

The beta and OMD approaches are useful rationalizations to the observer, but the real test is whether they offer anything to the operational investment manager. An assessment of their 'coalface' uses might include the following.

(i) *The zero capital framework*. Reporting requirements should include an assessment of the extent to which the current portfolio diverges from any benchmark, perhaps an externally imposed one. Casting the divergence as a proforma zero capital portfolio allows the direction and extent of the divergence to be quantified and reported.

(ii) *The beta approach*. This establishes a numerical text that any proposed enhancement portfolio must meet in order to qualify as acceptable. It also focuses attention on the sensitivities involved. For example, we saw that credit spread exposures differ materially according to just how they are accomplished. Credit derivatives have an inherently higher beta factor than do physicals. Estimated beta factors for the enhancement could also be made part of the reporting requirements.

(iii) *The OMD approach* . This removes some of the restrictive assumptions of the beta approach. It does not require the manager to know the precise value of the organisation's risk premium. In this respect, the horizontal axis of the OMD schedule has an important secondary role in constituting a spectrum of risk preferences (via the nodal P). The manager can square up the observed OMD values with his or her own preference area along the horizontal axis and in this way decide whether or not the proposed enhancement is likely to add to managerial or investor welfare. By plotting the OMD schedule for any proposed enhancement asset, it might be apparent at a glance whether or not the asset will be enhancing.

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