Pricing Access: Forward versus Backward Looking Cost Rules

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Abstract

Regulators across many different jurisdictions and industries have recently adopted the practice of setting access prices based on the current costs of providing the relevant facilities. Though widely regarded as being efficient, this practice has not been formally analyzed. Our analysis shows that given stochastic costs, forward looking access prices retard investment and are dominated by access prices determined by historic cost whenever investment is desired, unless the cost of investment is trending upwards with low volatility.

1 Introduction

There is a worldwide trend towards opening some parts of network industries to competition as a way of enhancing the welfare derived from what were usually state-owned monopolies. The price at which entrants can obtain access to the networks (such as access to electricity distribution or origination and termination of calls in the case of telecommunications), is a key determinant of the welfare gains secured by such pro-competitive policies. As a result, considerable attention is devoted to the design and implementation of access pricing regimes.

There is general agreement that in most standard networks access prices should be set based on the costs of the facilities provided,[1] but the definition of the relevant costs are a matter of some dispute. It has been argued, for example, that access prices should reflect the opportunity cost to the incumbent, including the lost profit, on the grounds that doing so prevents inefficient entry. This approach, often referred to as the efficient component pricing rule is discussed by Armstrong et al. (1996) and Laffont and Tirole (1994). This approach has considerable merit when retail prices are either regulated directly or constrained through effective competition. In all other cases it is only the direct cost of access, including a contribution to fixed costs, that should be allowed to enter the access price.

Recognising dynamic efficiency concerns, regulators typically set access prices at the long-run incremental cost of the service provided, where these cost measures allow for a reasonable return on capital outlays. The fact that most services depend on infrastructure that is common to the provision of other services requires that a method for attributing common costs be found. This problem

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[1] This assumes there are no network externalities which access prices need to internalize.

is generally approached by identifying the proportion of service life each asset devotes to the access service. The resulting measures are often referred to as the "total element (or service) long run incremental cost" (TELRIC or TSLRIC) of access.

It should be emphasised, however, that the TELRIC type label is usually also associated with a particular asset valuation methodology. This is by convention rather than necessity: in fact, any valuation method can be reflected in a TELRIC framework. Because our purpose is to study the effects of different valuation methods, we will minimise confusion by avoiding the use of TELRIC type labels.

Our analysis attempts to shed some light on the effect of different asset valuation methods. Though there are many such methods, almost all are based on one answer to a fundamental question: should assets be valued for regulatory purposes at the cost of the initial investment, or at the cost of re-building the facility at the present time?

Initially, regulators adopted the former approach, basing asset values on a company's historical accounts. For instance, in 1985 OFTEL, the UK telecommunications regulator, set access charges to British Telecom's network based on its historical costs (Melody, 2000, p. 274). We call the use of historical measures of asset values in calculating access prices "backward looking cost rules." This approach has been criticised by several authors, including Baumol and Sidak (1995) and Gans and Williams (1999), and has now been abandoned by many regulators.

The modern trend is towards the adoption of "forward looking cost rules," whereby the costs used to determine access prices are based on the current cost of rebuilding facilities to provide the existing service, using the best available technology. In 1994 OFTEL switched away from fully allocated costs based on historical accounts to a system based upon the computation of forward looking incremental costs reflecting current replacement costs of capital assets (Melody, 2000, p. 274). In fact, the regulatory authority in the U.S. (the FCC) proposed in its 1996 Telecommunications Act that the term "cost" should mean forward looking economic cost (Salinger, 1998, p. 150).

It is important to distinguish between the forward looking valuation of assets and the use of "optimisation" in valuing assets. Optimisation is a process by which assets are written out of the firm's valuation by the regulator, on the grounds that a new entrant would not require them. While frequently used in conjunction with forward looking asset valuation, optimisation is neither required in order to determine the current cost of the asset, nor restricted to forward looking valuation problems.

It is sometimes argued that forward looking access charges are desirable because they do not allow firms to recover inefficient investment. Clearly, however, this will only be so if optimisation is used, and since optimisation can also be applied to backward looking access charges, there is nothing special about forward looking charges in this regard. Moreover, because in practice optimisation requires numerous, often implausible, assumptions, its ability to eliminate inefficient investment is in any case overstated.

In this paper, we put aside such issues and focus on a more controversial argument in favour of forward looking cost rules. The proponents of forward looking access charges claim that competitors should not be stuck with an incumbent's high cost structure just because it invested at a time when costs were high.[2] In contestible markets, it is argued, a firm that tried to recover historical costs which are more than the current stand-alone cost of re-building the network that it provides, will be unable to do so. Thus, to mirror contestible markets, the costs included in access prices should be based on current best practice.

However, this misses an important point. The reason these markets are not contestable is firms need to sink large amounts of money into irreversible investments. Given uncertainty over the future cost of such projects, it is critical that they face the right incentives to do so in the first place. This paper addresses this point by asking if a firm, in a world of cost uncertainties, will invest earlier under a backward or forward looking cost rule and asking which rule leads to higher overall welfare.

We provide a model in which a firm has a single irreversible investment opportunity. The cost of carrying out the project varies stochastically through time. To focus on the effects of cost uncertainty, the (flow) return to the project is assumed constant for a given access price and there is no physical depreciation in the asset.[3] We assume the incumbent's profit is increasing in the access price charged at any time. Given an access pricing rule, the firm must decide when to invest.

With a fixed access price unrelated to the asset value, the firm will delay investment until the net present value of the project covers not only its costs, but also the option value of delaying investment. In doing so the firm waits too long from the regulator's point of view, since the firm ignores the surplus that would flow to competitors and consumers. What is needed is a means of encouraging the incumbent to invest earlier. Higher access prices would provide such an incentive, but higher access prices reduce the flow of surplus to competitors. This suggests there is a trade-off. High access charges lead to a flow of surplus that is low but starts sooner, while low access charges lead to a flow of surplus that is high but starts later. The preferred access pricing scheme will match the marginal cost of bringing investment further forward in time (the lower total surplus resulting from raising access charges) and the marginal benefit (earlier investment raises the present value of any given cashflow).

This trade-off can be improved by using either backward or forward looking access charges where these charges are expressed as a rate of return on asset values. Backward looking charges achieve this by making it more expensive for the incumbent to delay investment — by waiting for the cost of the project to fall, the incumbent ensures that the access charges will be lower as well. Thus, not only does waiting to invest delay the receipt of the profit flow, but it also reduces the level of the profit flow. Investment will therefore occur earlier under a backward looking rule than it would with a fixed access charge of the same size. This brings investment forward without raising the access charge, and thus without lowering the flow of total surplus. Forward looking rules also link access charges to the cost of the project, but to the project's replacement cost,

[2] The controversy arises from the fact that with costs falling over time, historical costs tend to be high and forward-looking costs low. This leads to an obvious conflict between access-seekers and access-providers over the appropriate methodology. Laffont and Tirole (2000, pp. 141–161) discuss the debate concerning backward versus forward looking cost-based pricing of access. Temin (1997) notes that as early as the 1960s, AT&T argued in favour of forward-looking cost approaches to justify low rates, while entrants supported historical cost measures to justify high rates.

[3] These assumptions imply that economic depreciation is also zero.

rather than its actual cost. Because the access charge under the forward looking rule diverges from the cost of the investment over time, it subjects the firm to unnecessary risk. To achieve the same investment decision with forward looking rules requires that the rate of return on capital be increased to compensate for the additional risk that forward looking rules impose. Since this is costly to consumers and access-seekers, the backward looking rule generally dominates in welfare comparisons.[4]

The advantages of backward looking rules over forward looking rules are stronger when there is a downward drift in costs. Whether access charges are backward looking or forward looking, the firm has an incentive to wait and invest when the cost of doing so is lower. This incentive to delay is weakened under a backward looking regime, because if the firm invests sooner it can lock-in a higher access charge, and hence raise the present value of its profit flow. In contrast, under a forward looking regime, investing when the cost is high guarantees high access charges in the short-run, but that is all. Future access charges are unaffected by the timing of investment. Investing early raises the present value of its profit flow, but not to the same extent as under a backward looking rule. With sufficient upward drift in costs this argument is reversed, and firms will have a stronger incentive to invest early under a forward looking rule. In this case, under both rules the firm is reluctant to wait, since the cost of the project is expected to increase over time. However, under a backward looking rule, waiting will allow the firm to lock-in a higher access charge. Under a forward looking rule, investing early also raises the present value of the firm's profit flow, but not to the same extent.

One situation when forward looking rules could dominate in a welfare comparison would be if the incumbent firm operated in the downstream market and was allowed through a backward looking cost rule to set such high access prices so as to allow it to monopolise this market. In this case, an increase in access price volatility can be socially beneficial. Higher access prices do not affect profits or surplus, since the firm remains a monopoly in its downstream market. However, lower access prices can increase surplus (by allowing for competition) with little reduction in the incumbent's profit.[5] A forward looking rule can deliver this increased volatility. (But note that the above case, and that of strong upward drift in costs, are not the typical cases in industries where forward looking costs rules are applied, such as in telecommunications.)

Our work is related to some recent research which takes a forward looking rule as given and considers how the rental rate on capital should be determined. Ergas and Small (2000) explore the relationship between economic depreciation and the value of the delay option in the context of regulated access. They establish conditions under which expected economic depreciation is identical to the value of the option to delay investment, and show that as a general matter the former (economic depreciation) is no less than the latter (the real option value). Salinger (1998) shows that the potential for competition, asset life

[4] Moreover, other things equal, greater cost volatility raises the volatility induced by forward looking rules. This strengthens the case against forward looking access rules.

[5] Essentially, the convexity of the regulator's surplus function around very high access charges more than offsets the corresponding concavity of the firm's profit function. Where the access provider does not compete with access-seekers, any convexity in the surplus function is not likely to occur unless access prices are at such high levels that the downstream market no longer exists (so surplus becomes zero). Such high access prices would never be allowed to occur under regulation since they lower both profits and surplus to zero.

uncertainty, and the installation of excess capacity for demand growth all raise the foward looking access price. For instance, when firms build projects they typically invest in excess capacity to meet potential future growth in demand, thus avoiding having to come back and add small increments to the initial infrastrucure (which would be inefficient). To the extent that forward looking rules ignore these additional costs, competitors are getting a real option which they are not paying for — the option to use the excess capacity if and when needed. If the competitors do not pay for this option, then the incumbent will invest too little in such excess capacity. Offsetting these effects he finds that, the potential for technological change that enhances the future value of an asset lowers forward looking costs. However, none of these papers addresses how forward looking access charges perform under stochastic costs, the feature which leads forward looking costs to give fundamentally different incentives to backward looking cost rules. Moreover, none of the papers attempts to evaluate the desirability of the forward looking approach.

The rest of the paper proceeds as follows. Section 2 sets up the firm's investment problem. In Section 3 the optimal investment policy is characterized for the constant, backward, and forward looking rules. The desirability of backward and forward looking rules over a constant access price is explained in Section 4. This section also provides the conditions under which the backward looking rule leads to earlier investment than the forward looking rule. Section 5 contains a welfare comparison of the two rules, while Section 6 concludes with a summary of results, policy implications and directions for future research.

2 Setting up the Model

A project, which can be launched at any time, involves a single, large, irreversible investment. If the project is launched at time t, it costs K_t.

Assumption 1 *The cost of launching the project evolves according to the geometric Brownian motion* $dK_t = \nu K_t dt + \sigma K_t d\xi_t$*, where* ν *and* σ *are constants and* $d\xi_t$ *is the increment of a Wiener process.*

We value all cashflows using contingent claims analysis. This is made possible by assuming that capital markets are sufficiently complete that we can construct a portfolio of traded assets whose value is perfectly correlated with the cost of launching the project.[6]

Assumption 2 *There exists an asset, or a portfolio of traded assets, whose price* X_t *evolves according to*

$$dX_t = \mu X_t dt + \sigma X_t d\xi_t,$$

where $\mu > \nu$ *is a constant. We assume that this portfolio pays no dividend.*

The principal advantage of this assumption is that it eliminates the need to model the appropriate risk-adjusted discount rate. Cashflow streams can instead

[6] If the cost K_t is not spanned by traded assets, we can use dynamic programming in place of contingent claims analysis. The form of our results will be the same, but they will involve a subjective discount rate. The disadvantage of such an approach is that, without making restrictive assumptions regarding investors' preferences, there is no way of determining the correct discount rate.

be valued by simply assuming that there are no arbitrage opportunities. The following result shows the consequence of this no-arbitrage assumption.[7]

Lemma 1 *Suppose an asset generates a continuous cashflow at rate $f_{t+s} = f(K_{t+s})$ for all $s \geq 0$, for some function f. Then the value of this cashflow at time t is $F(K_t)$, for some function F satisfying the ordinary differential equation*

$$0 = \frac{1}{2}\sigma^2 K^2 F''(K) + (r - \eta)KF'(K) - rF(K) + f(K),$$

where $\eta = \mu - \nu > 0$ and r is the riskless interest rate.

The regulator wants a private firm, which we call the incumbent, to construct the project. It will impose a regulatory framework specifying the access price which the regulator can charge a competitor for use of the facility. We suppose that launching the project initiates an indefinite flow of profit $\pi_t = \pi(a_t)$ to the incumbent, the level of which depends on the access price a_t.

Assumption 3 *The incumbent's profit flow $\pi(a)$ is positive, bounded above, increasing and concave in a, and $\pi'(0)$ is finite.*

When the access charge is zero, the competitor is able to use the incumbent's facility without charge. Higher access charges increase the incumbent's profit flow, but at a decreasing rate.[8] In the limit when the access charge is infinite, the competitor will choose not to participate, leading to the monopoly outcome. Thus, $\lim_{a\to\infty} \pi(a) = \pi_m$ is the monopoly level of the incumbent's profit flow.

Once the project has been launched, the regulator observes a flow of total surplus $\theta_t = \theta(a_t)$ which also depends on the access charge.

Assumption 4 *The regulator's flow of total surplus $\theta(a)$ is decreasing in a, with $\theta(a) > \pi(a)$ for all a.*

The regulator chooses the charging regime, while the incumbent takes this charging regime as given and chooses its investment policy. The regulator's aim is to select the charging regime which, given the incumbent's response, leads to the greatest possible present value of all future surpluses.

3 Optimal Investment Policy

This section explores the investment behavior of an incumbent faced with various access charging regimes. We begin by describing the optimal investment policy for a very general investment payoff function, before concentrating on three specific charging regimes.

We denote the payoff to the incumbent at the time of investment by $P(K)$, where K is the cost of launching the project. The payoff will equal the present value of the profit flow initiated by investment, less the cost of launching the project. The precise form of $P(K)$ will depend on the access charging regime in place.

[7] The proof of Lemma 1, together with the proofs of all other results, is contained in the appendix.

[8] A rationale for concave profits is that the incumbent does not get to optimise volume in this model; quite the reverse, entrants will use a lot when times are good. This reverses the usual (Jorgenson) assumption that maximum profit flows are convex in prices.

Suppose that the incumbent decides to invest whenever the cost of doing so is less than some critical level $\hat{K}$.

Lemma 2 *If the project has not already been launched at time t, the value of the incumbent's entitlement to the project at that time is*

$$V(K_t;\hat{K}) = \begin{cases} P(\hat{K})\left(\dfrac{\hat{K}}{K_t}\right)^{\gamma} & \text{if } K_t \geq \hat{K}, \\ P(K_t) & \text{if } K_t < \hat{K}, \end{cases}$$

where

$$\gamma = \frac{r-\eta}{\sigma^2} - \frac{1}{2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{r-\eta}{\sigma^2} - \frac{1}{2}\right)^2} > 0.$$

The incumbent's optimal investment policy is therefore to choose the investment threshold $\hat{K}$ which maximizes $P(\hat{K})\hat{K}^{\gamma}$. The following result describes the optimal investment threshold.

Lemma 3 *The optimal investment policy is to invest whenever the cost of doing so is less than the threshold $\hat{K}$ given implicitly by*

$$\frac{\hat{K}}{P(\hat{K})} \cdot \frac{dP(\hat{K})}{d\hat{K}} = -\gamma.$$

Now we are ready to start analyzing the relationship between the access charging regime imposed by the regulator and the investment behavior of the incumbent.

Under the simplest regime, the regulator sets a fixed access charge a. Since this leads to a constant profit flow $\pi(a)$, the present value of future profits, measured at the time the project is launched, is $\pi(a)/r$. In this case the incumbent's payoff from investment is

$$P_c(\hat{K}) = \frac{\pi(a)}{r} - \hat{K}.$$

Under backward looking (BL) access charges, the level of the access charge depends on the amount paid by the incumbent to launch the project: if the incumbent launches the project at time t, paying K_t, then the competitor must pay the incumbent $a_{t+s} = \rho K_t$ for all subsequent times $t+s$, for some constant ρ chosen by the regulator. Like the constant access charge, the access charge is still constant through time, but now it is not completely determined by the regulator. In fact, by relaxing the investment threshold (that is, raising $\hat{K}$), the incumbent can increase the access charge paid by the competitor, and thus increase its own profit flow. Suppose the incumbent chooses the investment threshold $\hat{K}$ when faced with the BL rule parameterized by ρ. Since the access charge equals $\rho\hat{K}$ throughout the life of the project, the present value of the profit flow equals $\pi(\rho\hat{K})/r$, and the payoff to investment is

$$P_b(\hat{K}) = \frac{\pi(\rho\hat{K})}{r} - \hat{K}.$$

Under a forward looking (FL) regime, if the project is launched at time t, the competitor pays $a_{t+s} = \rho K_{t+s}$ at time $t+s$ for all $s > 0$. It is easy to show, using Lemma 1, that the value of this cashflow at time t is $\rho K_t/\eta$.[9] However, when evaluating the project, the incumbent focuses on the value of the resulting profit flow, not the value of the flow of access charges.

Lemma 4 *Under a FL access charge, the present value of the incumbent's future profit flow equals* $\Pi_f(\rho K_t)$, *where*

$$\Pi_f(a) = \frac{1}{r} \cdot \frac{\gamma\delta}{\gamma+\delta}\left(\int_0^1 y^{\gamma-1}\pi(ay)dy + \int_0^1 y^{\delta-1}\pi(a/y)dy\right)$$

and

$$\delta = -\frac{r-\eta}{\sigma^2} + \frac{1}{2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{r-\eta}{\sigma^2} - \frac{1}{2}\right)^2} > 1.$$

If the incumbent chooses the investment threshold $\hat{K}$, the payoff to investment equals

$$P_f(\hat{K}) = \Pi_f(\rho\hat{K}) - \hat{K},$$

where Π_f is the function described in Lemma 4.

The precise optimal threshold for each regime can be found by substituting the appropriate payoff function into Lemma 3.

Proposition 1

1. *When faced with the regime with constant access charges* a, *the incumbent chooses the investment threshold* $\hat{K}_c = R_c(a)$, *where*

$$R_c(a) = \frac{\gamma}{\gamma+1} \cdot \frac{\pi(a)}{r}.$$

2. *When faced with the BL regime with implicit rental rate* ρ, *the incumbent chooses the investment threshold* $\hat{K}_b$ *given implicitly by* $\hat{K}_b = R_b(\rho\hat{K}_b)$, *where*

$$R_b(a) = \frac{\gamma}{\gamma+1} \cdot \frac{\pi(a)}{r} + \frac{1}{\gamma+1} \cdot \frac{a\pi'(a)}{r}.$$

3. *When faced with the FL regime with implicit rental rate* ρ, *the incumbent chooses the investment threshold* $\hat{K}_f$ *given implicitly by* $\hat{K}_f = R_f(\rho\hat{K}_f)$, *where*

$$R_f(a) = \frac{\gamma}{\gamma+1} \cdot \frac{\pi(a)}{r} + \frac{\gamma}{\gamma+1} \cdot \frac{a}{r}\int_0^1 y^{\delta-2}\pi'(a/y)dy.$$

4 Investment Behavior

In this section, we focus attention on the timing of investment under the three regimes introduced in the preceding section. The first result is familiar from real option theory. Since the incumbent has the option to delay investment, investing as soon as net present value is nonnegative is not optimal. Instead, the incumbent should wait until the investment payoff exceeds the value of the delay option destroyed by investment.

[9] The access charges start at the level ρK_t, are expected to grow at rate ν, and are discounted at rate μ. The value of all future access charges is therefore $\rho K_t/(\mu-\nu) = \rho K_t/\eta$.

Proposition 2 *Under all three access pricing regimes, the incumbent delays investment past the break-even point. That is,* $P_c(\hat{K}_c)$*,* $P_b(\hat{K}_b)$ *and* $P_f(\hat{K}_f)$ *are all strictly positive.*

In all three regimes, the advantage of bringing investment forward in time is that any given investment payoff will be discounted less, and as a result will be more valuable. However, the only way to invest earlier is to raise the investment threshold, leading to a greater investment outlay, and therefore a lower payoff to investment. This disadvantage is shared by all three regimes. However, its effects are reduced somewhat in the case of BL and FL regimes. Under the constant access charging rule, the present value of future profit flows is fixed; relaxing the investment threshold only increases the cost of launching the project. However, under BL and FL rules, relaxing the investment threshold also increases the access charges received by the incumbent, increasing the present value of the profit flow, and thereby offsetting the increased cost of launching the project.[10] Thus, under BL and FL costs, the incumbent finds it less expensive to bring investment forward in time; that is, BL and FL rules motivate the incumbent to invest sooner.

Proposition 3 *BL and FL regimes lead to earlier investment than constant access charging regimes generating profit flows having the same present value. That is, for any implicit rental rate* ρ*,*

1. *If the constant access charge is such that* $\pi(a)/r = \pi(\rho\hat{K}_b(\rho))/r$*, then* $\hat{K}_c \leq \hat{K}_b(\rho)$*.*
2. *If the constant access charge is such that* $\pi(a)/r = \Pi_f(\rho\hat{K}_b(\rho))$*, then* $\hat{K}_c \leq \hat{K}_f(\rho)$*.*

FL costs share the most important of the BL rule's features — by investing earlier, the incumbent receives a higher access charge — but the two regimes differ in one important respect. With BL costs, a high access charge is locked in. Under FL costs, if the replacement cost of the project falls, so does the access charge. As the next result shows, provided the drift in cost is not too large, investment occurs sooner under a BL rule than under a FL rule with the same implicit rental rate.

Proposition 4 *If* $\nu \leq \mu - r$*, any BL rule leads to earlier investment than the FL rule with the same rental rate.*

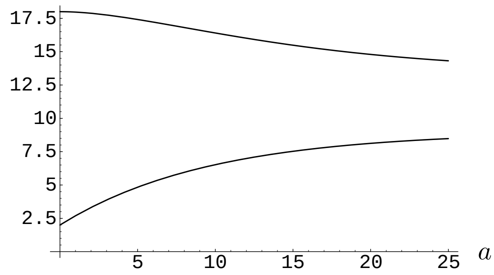
That is, if the project's replacement cost is expected to grow at a rate ν no higher than the risk-premium $\mu - r$, investment occurs sooner under a BL regime than under a FL regime with the same rental rate. In particular, if the cost has nonnegative systematic risk, BL costs will lead to earlier investment whenever the cost of launching the project is expected to fall over time.

If $\nu \leq \mu - r$ and the incumbent chose the same investment threshold for FL charges as for BL ones with the same implicit rental rate, the present value of the FL charges would be lower than for the BL ones.[11] The payoff to investment

[10] This is obvious in the case of BL costs, since there the access charges are constant through time, but not so obvious in the case of FL costs, where access charges fluctuate over time. However, it is clear from Lemma 4 that $\Pi_f(\rho\hat{K})$ is an increasing function of $\hat{K}$.

[11] If the common investment threshold is $\hat{K}$, the present value of access charges under BL costs is $\rho\hat{K}/r$, and under FL costs is $\rho\hat{K}/\eta$.

Figure 1: Profit and total surplus flows



The bottom curve plots the profit flow $\pi(a) = 9 - 7 \times 2^{-0.15a}$, while the top curves plot the total surplus flow $\theta(a) = 13.5 + 4.5(1 + a/8)e^{-a/8}$.

would be lower under FL costs than under BL costs with the same investment threshold. This encourages the incumbent to choose a lower threshold, spending less on launching the project, under the FL regime than under the BL regime.

However, the trend in the project's replacement cost is just part of the story behind Proposition 4, for the result also holds when $\nu = \mu - r$. In this case, the volatility of FL access charges, combined with the concavity of the incumbent's profit flow function, means that the investment payoff will still be lower under the FL regime, when a common investment threshold is chosen under both rules. Once more, under FL costs the incumbent is motivated to wait until launching the project is cheaper.

The issue of volatility remains when $\nu > \mu - r$. Although FL access charges will have greater value than BL ones (for the same investment threshold), the concavity of the incumbent's profit flow function lowers the value of the FL access charges to the incumbent. The effect will be greater for high values of σ. Thus, when the drift in replacement cost is high, which of the two rules leads to earlier investment will depend on both the drift and volatility of the replacement cost.

Proposition 5 *There exists a function $M(\cdot, \rho)$, bounded above by 1, such that the BL rule with rental rate ρ leads to earlier investment than the FL rule with the same rental rate if and only if*

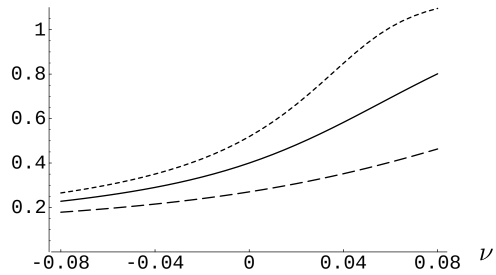
$$\nu \leq \mu - rM\left(\frac{\sigma^2}{r}, \rho\right).$$

Further investigation of the incumbent's response to BL and FL rules requires the use of numerical analysis. For our analysis, we adopt the profit flow function

$$\pi(a) = 9 - 7 \times 2^{-0.15a}.$$

It is drawn as the upward-sloping function in Figure 1. (The downward-sloping curve, which plots the regulator's total surplus flow, will be used in Section 5.) Consistent with Assumption 3, profit flow is increasing and concave in the access charge. Profit flow ranges from the free-access level $\pi(0) = 2$ to the monopoly

Figure 2: Investment timing under rules with $\rho = \mu$



Each curve plots the investment threshold under FL costs as a proportion of its value under BL costs as a function of the drift in costs. The dotted curve corresponds to low volatility ($\sigma = 0.1$), the solid curve to moderate volatility ($\sigma = 0.3$), and the dashed curve to high volatility ($\sigma = 0.5$).

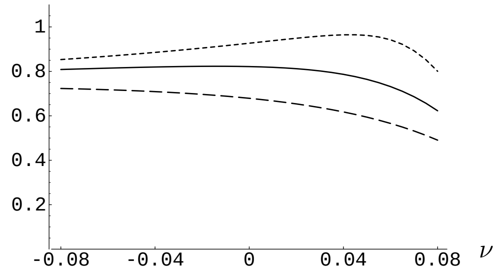
level $\lim_{a\to\infty} \pi(a) = 9$. The average of these two extremes occurs when the access charge equals 6.67. In all the cases we consider, the risk-adjusted discount rate is $\mu = 0.10$, and the riskless interest rate is $r = 0.05$.

Figure 2 compares the incumbent's investment behavior under BL and FL rules in the benchmark case where the regulator sets the implicit rental rates equal to the risk-adjusted discount rate μ. The graph plots the value of the investment threshold under FL costs as a proportion of its value under BL costs (that is, $\hat{K}_f(\mu)/\hat{K}_b(\mu)$) as a function of the drift in costs. Wherever the function is less than 1, the BL rule leads to earlier investment than the FL rule; wherever the function is greater than 1, the BL rule leads to later investment than the FL rule. The dotted curve corresponds to low volatility ($\sigma = 0.1$) in the cost of launching the project, the solid curve to moderate volatility ($\sigma = 0.3$), and the dashed curve to high volatility ($\sigma = 0.5$). With moderate to high volatility in costs, the BL regime leads to earlier investment than the FL regime, regardless of the drift in costs. However, when volatility is low, the drift in costs is a crucial determinant of the relative performance of the two rules: when drift is negative, FL costs retard investment, while when drift is large and positive, FL costs actually lead to earlier investment than the benchmark BL rule.

When volatility is low, the main difference between the two regimes is that the access charge is constant under BL costs, and changing over time under FL costs. If drift is negative, then the present value of FL access charges will be lower, relative to the cost of launching the project, than BL access charges, and the incumbent will delay investment longer under FL costs than under BL costs. However, when the trend in FL access charges is positive, the resulting high present value will encourage the incumbent to invest earlier under FL costs than under BL costs.

We investigate this further by neutralizing the drift factor. Figure 3 compares the performance of BL and FL rules when the regulator sets the implicit rental rates such that the present value of the access charges equals the cost of launching the project. In effect, the rental rate is adjusted for the risk of the access charges, as well as any trend. This is achieved by setting $\rho_b = r$ and

Figure 3: Investment timing under rules with $\rho_b = r$ and $\rho_f = \eta$



Each curve plots the investment threshold under FL costs as a proportion of its value under BL costs as a function of the drift in costs. The dotted curve corresponds to low volatility ($\sigma = 0.1$), the solid curve to moderate volatility ($\sigma = 0.3$), and the dashed curve to high volatility ($\sigma = 0.5$).

$\rho_f = \eta$. In all situations considered, the BL leads to earlier investment. Once we take away the advantage to FL costs offered by the positive trend in access charges, BL costs again dominate in all the situations considered.

Why do FL costs retard investment here? With the rental rates used in Figure 3, the competitors effectively pay for the entire project through the access charges. The principal difference between the two regimes is that under FL costs, the flow of access charges, and hence the incumbent's profit flow, is stochastic. This key difference explains why investment occurs sooner under BL costs. As is familiar from real option theory, greater uncertainty in the cost of launching the project will make the incumbent delay investment longer. Here, however, the effect of higher volatility is more severe when the regulator imposes FL costs than when it imposes BL costs. This is evident in both Figures 2 and 3, where higher volatility causes the ratio $\hat{K}_f/\hat{K}_b$ to fall; that is, the investment threshold under the FL regime falls more than the investment threshold under the BL regime. This situation arises because the incumbent's profit flow is concave in access charges. Even though the present value of access charges equals the cost of the project under both rules, the volatility in FL access charges means that the present value of the profit flow will be lower under FL costs. The incumbent must wait longer, in order to pay less for the project, to compensate for this. Hence, FL costs discourage investment.

5 Welfare Analysis

Up until this point, our attention has focused on the timing of the incumbent's investment decision under the three regimes. This section compares the various charging regimes from the regulator's point of view. In Section 5.1 we describe how the regulator assesses the three charging regimes introduced in Section 3. We present and analyze the regulator's problem graphically in Section 5.2, before performing a more detailed numerical analysis in Section 5.3.

5.1 Evaluating Charging Regimes

We let $S(K)$ denote the payoff to the regulator at the time of investment, where K is the cost of launching the project. We interpret $S(K)$ as the present value of the flow of total surplus, less the cost of launching the project. The form this function takes depends on the access charging regime imposed by the regulator.

Under a constant access charging regime, the regulator observes a constant flow of total surplus equal to $\theta(a)$, where a is the level of the access charge. This stream has present value $\theta(a)/r$, implying that the regulator's payoff function is

$$S_c(K) = \frac{\theta(a)}{r} - K.$$

Under a BL regime, the access charge is also constant through time. If the implicit rental rate equals ρ, the access charge equals ρK. The constant flow of total surplus therefore has present value $\theta(\rho K)/r$, so that the regulator's payoff function is

$$S_b(K) = \frac{\theta(\rho K)}{r} - K.$$

The construction of the regulator's objective function is less straightforward under FL costs.

Lemma 5 *Under a FL access charge with implicit rental rate ρ, the regulator's payoff function is*

$$S_f(K) = \Theta_f(\rho K) - K,$$

where

$$\Theta_f(a) = \frac{1}{r} \cdot \frac{\gamma\delta}{\gamma+\delta} \left(\int_0^1 y^{\gamma-1}\theta(ay)dy + \int_0^1 y^{\delta-1}\theta(a/y)dy \right).$$

At any given time, the value of the regulator's future flow of total surplus will depend on the current cost of launching the project, the investment threshold chosen by the incumbent, and the exact form of S.

Proposition 6 *If the project has not already been launched at time t, and the incumbent has chosen the investment threshold $\hat{K}$, then the net present value of the future surpluses at that time is*

$$W(K_t; \hat{K}) = \begin{cases} S(\hat{K}) \left(\dfrac{\hat{K}}{K_t} \right)^{\gamma} & \text{if } K_t \geq \hat{K}, \\ S(K_t) & \text{if } K_t < \hat{K}. \end{cases}$$

1. *When faced with the regime with constant access charges a, the regulator uses the payoff function*

$$S_c(K) = \frac{\theta(a)}{r} - K.$$

2. *When faced with the BL regime with implicit rental rate ρ, the regulator uses the payoff function*

$$S_b(K) = \frac{\theta(\rho K)}{r} - K.$$

3. *When faced with the FL regime with implicit rental rate ρ, the regulator uses the payoff function*

$$S_f(K) = \Theta_f(\rho K) - K.$$

When evaluating different access pricing schemes, the regulator uses the objective function $S(\hat{K})\hat{K}^{\gamma}$.

5.2 Representing the Regulator's Problem Graphically

In this section, we use Figure 4 to highlight the most important features of the problem facing the regulator. We focus on constant access charges to make the exposition as clear as possible, but the insights gained also apply to the more complicated BL and FL regimes.

The straight line through points A and C in Figure 4 plots combinations $(\hat{K}, S)$ resulting from a regime where the competitor is granted free access to the project. That is, it represents the line

$$S(\hat{K}) = \frac{\theta(0)}{r} - \hat{K}.$$

The dashed curves plot various level curves of $S\hat{K}^{\gamma}$ (or, equivalently, $P\hat{K}^{\gamma}$). If the regulator could choose the investment threshold, it would choose the threshold so that the outcome is at point A. This is the first-best solution to the access pricing problem. Like the incumbent, the regulator would wait for the cost to fall below some threshold before investing. However, as we shall see, the incumbent is generally too 'patient' for the regulator's liking. This is chiefly because the incumbent ignores the payoff to customers when evaluating the investment payoff.

In practice, the regulator does not choose the investment threshold. Suppose that, as in the first-best case, the regulator allows the competitor free access to the facility. The incumbent then maximizes its objective function subject to the constraint represented by the straight line through point B, which describes the incumbent's payoff function $P_0(\hat{K}) = \pi(0)/r - \hat{K}$. The optimal investment threshold is determined by the point of tangency with the incumbent's indifference curve, labelled B in Figure 4. The first-best threshold is easily shown to be

$$\frac{\gamma}{\gamma+1} \cdot \frac{\theta(0)}{r},$$

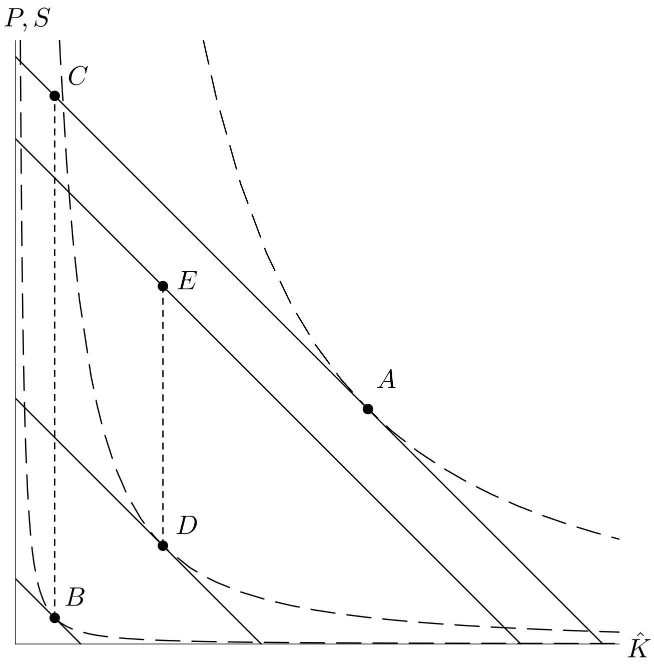
whereas the incumbent chooses

$$\frac{\gamma}{\gamma+1} \cdot \frac{\pi(0)}{r}.$$

From Assumption 4, $\theta(0) > \pi(0)$, so that the incumbent delays investment too long for the regulator's liking. From the regulator's point of view, the outcome is represented by point C in Figure 4 — the investment payoff is much higher than in the first-best case (reflecting the incumbent's greater profit), but will be received too late.

Suppose the regulator imposes a positive access charge a. Since the profit flow function is increasing, and the flow of total surplus is decreasing, in the

Figure 4: Representing the regulator's problem graphically



The horizontal coordinate is the investment threshold. The vertical coordinate is the investment payoff (P for the incumbent, and S for the regulator).

access charge, the incumbent's constraint moves outwards to the straight line through point D and the regulator's 'constraint' moves in to the straight line through point E. As shown in Figure 4, this induces the incumbent to choose a higher investment threshold (point D, compared with B when the competitor had free access to the project). From the regulator's point of view, the outcome moves from point C to E. This reduction in payoff offsets the improvement in welfare resulting from the earlier investment.

Figure 4 illustrates the problem faced by the regulator. With the competitor provided with free access to the facility, the incumbent is too reluctant to invest. By introducing a positive constant access charge, the regulator is able to induce the incumbent to invest sooner. The cost of this strategy is that, because the regulator's total surplus is a decreasing function of access charges, its payoff from investment will fall. One can imagine tracing out a curve, parameterized by the access charge a, which passes through points C and E. The optimal access charge would correspond to the point on this curve where the value of the regulator's objective function is greatest. At this point, the marginal cost of raising the access charge any higher would exactly match the marginal benefit resulting from the ensuing earlier investment.

The regulator faces similar difficulties with BL and FL charging regimes. The challenge is to induce the incumbent to invest sooner, without setting access charges so high that the resulting drop in the present value of the flow of total surplus cancels out the benefits of earlier investment. In fact, the use of

BL charges makes this task easier. Recall from Proposition 3 that the incumbent invests sooner under a BL regime than under a constant access charging regime with the same access charge. Therefore, simply by switching from constant access charges to a particular BL access charge, the regulator can bring investment forward in time without affecting the level of the access charge, and hence the present value of total surplus. This leads to the following result:

Proposition 7 *Each constant access charging rule is dominated by at least one BL charging rule.*

We now turn to the comparison of BL and FL rules from the regulator's perspective.

5.3 Numerical Analysis

We adopt a similar approach to that in Section 4. The difference is that in Section 4 we were interested in the timing of investment by the incumbent, whereas now we compare the various charging regimes from the point of view of the regulator. We start by comparing BL and FL rules with implicit rental rates equal to the risk-adjusted discount rate, and then compare rules with rental rates chosen such that the present value of the access charges for each rule equals the cost of launching the project. Finally, we compare the performance of the best BL rule with that of the best FL rule, and investigate the properties of the optimal rental rates for both BL and FL rules.

We use the total surplus flow function shown in Figure 1:

$$\theta(a) = 13.5 + 4.5\left(1 + \frac{a}{8}\right)e^{-a/8}.$$

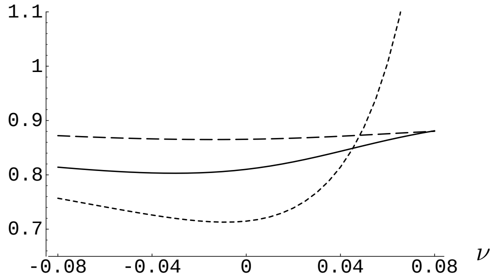
It is decreasing on $[0, \infty)$, concave on $[0, 8]$, and convex on $[8, \infty)$. The flow of total surplus equals 18 when the competitor is allowed free access to the project, and falls to 13.5 in the monopoly outcome.

Figure 5 compares the performance of BL and FL rules in the benchmark case where the regulator sets the implicit rental rates equal to the risk-adjusted discount rate μ. The graph plots the value of the regulator's objective function under FL costs as a proportion of its value under BL costs as a function of the drift in costs — the height of each curve is

$$\frac{S_f(\hat{K}_f(\mu))(\hat{K}_f(\mu))^\gamma}{S_b(\hat{K}_b(\mu))(\hat{K}_b(\mu))^\gamma}.$$

Wherever the function is less than 1, the BL rule dominates the FL rule from a welfare perspective; wherever the function is greater than 1, the FL rule dominates. The dotted curve corresponds to low volatility ($\sigma = 0.1$) in the cost of launching the project, the solid curve to moderate volatility ($\sigma = 0.3$), and the dashed curve to high volatility ($\sigma = 0.5$). With moderate to high volatility in costs, the drift in costs has little effect on the relative performance of the two rules, with the BL rule easily dominating the FL rule. However, when volatility is low, the drift in costs is a crucial determinant of the relative performance of the two rules: when drift is negative, FL costs perform particularly poorly, while when drift is large and positive, FL costs actually dominate the benchmark BL rule.

Figure 5: Welfare assessment of rules with $\rho = \mu$



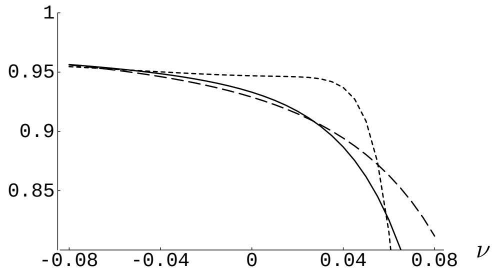
Each curve plots the value of the regulator's objective function under FL costs as a proportion of its value under BL costs as a function of the drift in costs. The dotted curve corresponds to low volatility ($\sigma = 0.1$), the solid curve to moderate volatility ($\sigma = 0.3$), and the dashed curve to high volatility ($\sigma = 0.5$).

Qualitatively, at least, these results are similar to those for investment timing reported in Figure 2. There we found that the benchmark BL rule generated earlier investment than the corresponding FL rule, except in the special case where the project's replacement cost was trending upwards with little volatility. However, close inspection of the two figures shows some differences. For example, when the drift in cost is negative Figure 2 shows that the FL investment threshold is substantially lower than the BL one, indicating that investment would be delayed quite some time. However, from the point of view of welfare, this is offset by the negative drift in access charges — FL access charges will fall over time, and the flow of total surplus will rise. Thus, although the FL rule is inferior to the BL one when drift is negative, the difference is not as great as the investment behavior evident in Figure 2 might suggest. Similarly, when the drift in replacement cost is positive (and volatility is low), the benefit of earlier investment under FL costs is partially offset by the negative trend in the flow of total surplus caused by rising access charges. Nevertheless, the timing of investment, rather than the trend in access charges, seems to be the dominant factor.

We investigate this further by neutralizing the drift factor as we did in Section 4. Figure 6 compares the performance of BL and FL rules when the regulator sets the implicit rental rates such that the present value of the access charges equals the cost of launching the project; that is, $\rho_b = r$ and $\rho_f = \eta$. With this adjustment, the BL is clearly superior from a welfare perspective. Across a range of volatilities, the relative performance of the FL rule deteriorates as drift increases. Figure 3 shows that when drift is already high, increasing it further tends to delay investment longer under FL costs than under BL costs. This will contribute to the lower welfare under FL rules evident from Figure 6. However, the upward drift in FL access charges, and the resulting downward drift in the flow of total surplus, compounds the problem, and explains the steep decline in relative performance shown by Figure 6.[12]

[12]It is interesting to note that the FL rule suffers less from the increasing drift in cost when volatility is high. The reason is that the high drift leads to FL access charges rapidly reaching

Figure 6: Welfare assessment of rules with $\rho_b = r$ and $\rho_f = \eta$



Each curve plots the value of the regulator's objective function under FL costs as a proportion of its value under BL costs as a function of the drift in costs. The dotted curve corresponds to low volatility ($\sigma = 0.1$), the solid curve to moderate volatility ($\sigma = 0.3$), and the dashed curve to high volatility ($\sigma = 0.5$).

Finally, Figure 7 compares the performance of BL and FL rules when the regulator chooses the welfare-maximizing implicit rental rate in each case. For example, the rental rate for the BL rule is

$$\rho_b^* = \arg \max_{\rho} \; S_b(\hat{K}_b(\rho))(\hat{K}_b(\rho))^{\gamma}.$$

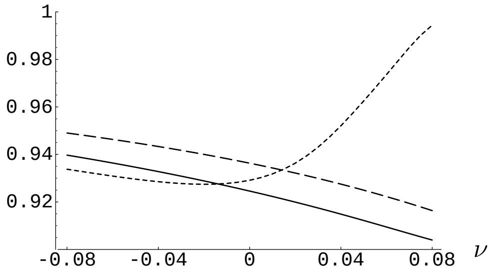
The optimal FL rental rate, ρ_f^*, is defined analogously. The figure shows that, except when the cost of the project is growing rapidly with little volatility, the best BL rule gives greater welfare than the best FL rule. In the usual case where the cost of the project is expected to fall over time, the BL rule is clearly preferred by the regulator.

Figure 8 illustrates the properties of the optimal implicit rental rates for BL and FL rules. The top three curves in the graph plot ρ_f^* as a function of μ, while the bottom three curves plot the optimal BL rental rate. In all cases considered, the optimal FL rental rate is higher than the optimal BL rental rate. Except when drift is high and volatility is low, the optimal FL rental rate is considerably higher than the optimal BL rental rate, which is only slightly less than the risk-adjusted discount rate ($\mu = 0.01$). The optimal BL and FL rental rates respond quite differently to changes in the drift and volatility of the cost of launching the project. The optimal BL rental rate is increasing in drift, and it is most sensitive to drift when volatility is low. The optimal BL rental rate is decreasing in volatility, and it is most sensitive when drift is high. The optimal FL rental rate is generally increasing in volatility and decreasing in drift. The exception occurs in low volatility, high drift situations.

The intuition for the behavior of the FL rental rate is clear. With FL costs, the incumbent is exposed to the risk that once the project is completed, the replacement cost (and hence the access charges) will fall. A high FL rental rate is required to compensate the incumbent for bearing this risk; without such

levels where the total surplus flow function is convex. More volatile cost, and hence more volatile access charges, therefore lead to a relatively high expected value for total surplus flow in the distant future. This helps offset the later investment and downward trend in the flow of total surplus.

Figure 7: Welfare assessment of rules with $\rho_b = \rho_b^*$ and $\rho_f = \rho_f^*$



Each curve plots the value of the regulator's objective function under FL costs as a proportion of its value under BL costs as a function of the drift in costs. The dotted curve corresponds to low volatility ($\sigma = 0.1$), the solid curve to moderate volatility ($\sigma = 0.3$), and the dashed curve to high volatility ($\sigma = 0.5$).

compensation, the incumbent will delay investment too long for the regulator's liking. When drift is positive, the risk is reduced, allowing the regulator to set a lower FL rental rate. When volatility is high, so is the risk to the incumbent, and the regulator is forced to set a higher rental rate.

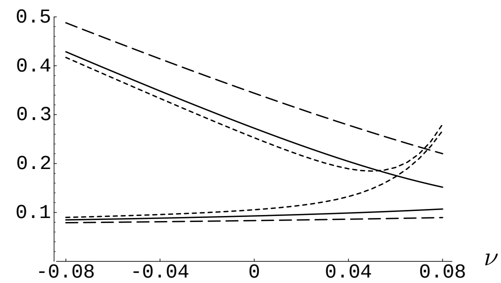
6 Conclusion

Using a simple model of an investment project, we found that when a regulator imposes a constant access charge a firm adopts an investment policy which is too conservative — it waits too long before investing. This inefficient delay arises because the firm bears all the cost of bringing investment forward in time, but shares the benefit of earlier investment with its competitors. However, by switching from fixed access charges to either backward looking or forward looking ones, the regulator can induce the firm to invest sooner without affecting the present value of the firm's profit flow. It achieves this by ensuring that the firm's competitors start to bear some of the cost of bringing investment forward in time.

Forward looking rules do so by allowing access prices to reflect current costs. Even if the current cost of launching the project is too high for investment to be optimal under a regime of fixed access charges, it may well be optimal to invest under a forward looking regime. The reason is that access prices are high in this situation, leading to a high profit flow. However, forward looking rules impose additional risk on the firm — it is exposed to future movements in costs once the investment costs have been sunk. This extra risk limits the extent to which investment can be encouraged.

Backward looking rules, which set access charges depending on the cost at the time of investment, are more successful at promoting investment. Like forward looking rules, they allow the firm to shift some of the cost of investing early onto its competitors. Unlike forward looking rules, they do not expose the firm to the risk of future movements in costs. In the usual case of downward

Figure 8: Behavior of optimal implicit rental rates



The curves plot the optimal rental rates under FL costs and under BL costs as a function of the drift in costs. The dotted curves correspond to low volatility ($\sigma = 0.1$), the solid curves to moderate volatility ($\sigma = 0.3$), and the dashed curves to high volatility ($\sigma = 0.5$). For each pair, the upper curve shows the optimal FL rental rate, and the lower curve the optimal BL rental rate.

(or no) drift in costs, there is no ambiguity — backward looking rules lead to earlier investment. Numerical examples suggest that for realistic cases they also dominate in terms of welfare. Even with some upward drift in costs, as long as volatility in costs is sufficiently great, backward looking rules still lead to earlier investment and higher welfare.

The policy implications of this paper are twofold. Firstly, except in the special situation where costs are climbing rapidly, and with little volatility, backward looking rules should be adopted. Secondly, if a forward looking rule is used, the implicit rental rate should be set at a level considerably higher than the risk-adjusted discount rate. A high rate is required to compensate the incumbent for the risk it bears when faced with forward looking access charges. Without such compensation, the incumbent will delay investment too long.

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Proofs

Proof of Lemma 1

Consider the portfolio made up of one unit of the asset being valued and $-K_tF'(K_t)/X_t$ units of the spanning asset. The value of this portfolio at time t equals

$$F(K_t) - K_tF'(K_t)$$

and, using Itô's Lemma, its value evolves according to

$$dF(K_t) - \frac{K_tF'(K_t)}{X_t}dX_t + f(K_t)dt = \left(\frac{1}{2}\sigma^2K_t^2F''(K_t) - (\mu-\nu)K_tF'(K_t) + f(K_t)\right)dt.$$

This portfolio is therefore riskless, and must earn the riskless rate of return r. Thus

$$\frac{1}{2}\sigma^2K_t^2F''(K_t) - (\mu-\nu)K_tF'(K_t) + f(K_t) = r(F(K_t) - K_tF'(K_t)),$$

and the ordinary differential equation for F follows immediately.

Proof of Lemma 2

Clearly, if $K \le \hat{K}$, $V(K;\hat{K}) = P(K)$, since investment occurs immediately. Suppose, instead, that $K > \hat{K}$, so that investment will be delayed for some unknown period. From Lemma 1, V satisfies the ordinary differential equation

$$0 = \frac{1}{2}\sigma^2K^2V_{KK} + (r-\eta)KV_K - rV.$$

The general solution to this equation is

$$V(K;\hat{K}) = C_1K^{-\gamma} + C_2K^{\delta},$$

where C_1 and C_2 are arbitrary constants and

$$\begin{aligned}
\gamma &= \frac{r-\eta}{\sigma^2} - \frac{1}{2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{r-\eta}{\sigma^2} - \frac{1}{2}\right)^2} > 0, \\
\delta &= -\frac{r-\eta}{\sigma^2} + \frac{1}{2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{r-\eta}{\sigma^2} - \frac{1}{2}\right)^2} > 1.
\end{aligned}$$

We require that $V(K;\hat{K}) \to 0$ as $K \to \infty$, so that the rights to the project are worthless when launching is prohibitively expensive. This forces $C_2 = 0$. Since investment in the project is triggered when K_t falls below $\hat{K}$, we must have $V(\hat{K};\hat{K}) = P(\hat{K})$. It follows that

$$V(K;\hat{K}) = P(\hat{K})\left(\frac{\hat{K}}{K}\right)^{\gamma}.$$

Proof of Lemma 4

From Lemma 1, the function Π_f must satisfy the ordinary differential equation

$$\frac{1}{2}\sigma^2 a^2 \Pi_f''(a) + \mu a \Pi_f'(a) - r\Pi_f(a) + \pi(a) = 0.$$

Since $a = 0$ is an absorbing barrier for geometric Brownian motion, we must have

$$\Pi_f(0) = \int_0^\infty e^{-rs}\pi(0)ds = \frac{\pi(0)}{r}.$$

It is straightforward to confirm that

$$\Pi_f(a) = \frac{1}{r} \cdot \frac{\gamma\delta}{\gamma+\delta}\left(a^{-\gamma}\int_0^a x^{\gamma-1}\pi(x)dx + a^{\delta}\int_a^\infty x^{-\delta-1}\pi(x)dx\right)$$

satisfies the ordinary differential equation above. Making the change of coordinate $y \mapsto x/a$ in the first integral and $y \mapsto a/x$ in the second shows that

$$\Pi_f(a) = \frac{1}{r} \cdot \frac{\gamma\delta}{\gamma+\delta}\left(\int_0^1 y^{\gamma-1}\pi(ay)dy + \int_0^1 y^{\delta-1}\pi(a/y)dy\right),$$

as required.

Proof of Proposition 1

We need to consider the case of FL access charges. The first order condition for the incumbent's maximization problem can be written in the form $\hat{K}_f = R_f(\rho\hat{K}_f)$, where

$$R_f(a) = \frac{\gamma}{\gamma+1} \cdot \Pi_f(a) + \frac{1}{\gamma+1}a\Pi_f'(a).$$

Integration by parts confirms that

$$a\int_0^1 y^{\gamma}\pi'(ay)dy = \pi(a) - \gamma\int_0^1 y^{\gamma-1}\pi(ay)dy$$

and

$$a\int_0^1 y^{\delta-2}\pi'(a/y)dy = -\pi(a) + \delta\int_0^1 y^{\delta-1}\pi(a/y)dy.$$

Therefore

$$\begin{aligned} R_f(a) &= \frac{\gamma}{\gamma+1} \cdot \Pi_f(a) + \frac{1}{\gamma+1}a\Pi_f'(a) \\ &= \frac{1}{\gamma+1} \cdot \frac{\gamma\delta}{\gamma+\delta} \cdot \frac{1}{r}\left(\gamma\int_0^1 y^{\gamma-1}\pi(ay)dy + \gamma\int_0^1 y^{\delta-1}\pi(a/y)dy \right. \\ &\qquad \left. + a\int_0^1 y^{\gamma}\pi'(ay)dy + a\int_0^1 y^{\delta-2}\pi'(a/y)dy\right) \\ &= \frac{\gamma\delta}{\gamma+1} \cdot \frac{1}{r}\int_0^1 y^{\delta-1}\pi(a/y)dy. \qquad (1) \end{aligned}$$

Integrating this expression by parts gives

$$
\begin{aligned}
R_f(a) &= \frac{\gamma\delta}{\gamma+1} \cdot \frac{1}{r} \int_0^1 y^{\delta-1} \pi(a/y) dy \\
&= \frac{\gamma}{\gamma+1} \cdot \frac{1}{r} \int_0^1 d(y^{\delta}) \pi(a/y) \\
&= \frac{\gamma}{\gamma+1} \cdot \frac{\pi(a)}{r} + \frac{\gamma}{\gamma+1} \cdot \frac{a}{r} \int_0^1 y^{\delta-2} \pi'(a/y) dy. \qquad (2)
\end{aligned}
$$

Proof of Proposition 2

The following result will be useful in the proofs of this and some subsequent results. The concavity of π implies that the function $\pi(a) - a\pi'(a)$ is increasing in a for all nonnegative a, so that

$$\pi(a) - a\pi'(a) \geq \pi(0) > 0.$$

Therefore

$$a\pi'(a)/\pi(a) < 1 \text{ for all } a \geq 0. \qquad (3)$$

With a constant access charge equal to a, the investment payoff, evaluated at the investment threshold, is

$$P_c(\hat{K}_c) = \frac{\pi(a)}{r} - \hat{K}_c = \frac{1}{\gamma+1} \cdot \frac{\pi(a)}{r} > 0.$$

With a BL rental rate of ρ, the investment payoff is

$$P_b(\hat{K}_b) = \frac{\pi(\rho\hat{K}_b)}{r} - \hat{K}_b = \frac{1}{\gamma+1} \cdot \frac{1}{r}(\pi(\rho\hat{K}_b) - \rho\hat{K}_b\pi'(\rho\hat{K}_b)).$$

The inequality (3) proves that the payoff is positive.

Finally, with a FL rental rate of ρ, the investment payoff is

$$
\begin{aligned}
P_f(\hat{K}_f) &= \Pi_f(\rho\hat{K}_f) - \hat{K}_f \\
&= \Pi_f(\rho\hat{K}_f) - R_f(\rho\hat{K}_f) \\
&= \frac{1}{r} \cdot \frac{\gamma\delta}{\gamma+\delta} \int_0^1 y^{\gamma-1} \pi(\rho\hat{K}_f y) dy,
\end{aligned}
$$

which is clearly positive.

Proof of Proposition 3

Consider the BL rule with implicit rental rate ρ and let $a = \rho\hat{K}_b(\rho)$. If the regulator imposed the constant access charge a, the incumbent would choose the investment threshold

$$\hat{K}_c = R_c(a) = \frac{\gamma}{\gamma+1} \cdot \frac{\pi(a)}{r} \leq \frac{\gamma}{\gamma+1} \cdot \frac{\pi(a)}{r} + \frac{1}{\gamma+1} \cdot \frac{a\pi'(a)}{r} = R_b(a) = \hat{K}_b(\rho).$$

This would lead to later investment, completing the proof of the first part of the proposition.

For the second part, consider the FL rule with implicit rental rate ρ and choose a such that

$$\pi(a) = r\Pi_f(\rho\hat{K}_f(\rho)).$$

If faced with this constant access charge, the incumbent would choose the investment threshold

$$\begin{aligned}\hat{K}_c &= \frac{\gamma}{\gamma+1}\cdot\frac{\pi(a)}{r} \\ &= \frac{\gamma}{\gamma+1}\Pi_f(\rho\hat{K}_f(\rho)) \\ &\leq \frac{\gamma}{\gamma+1}\cdot\Pi_f(\rho\hat{K}_f(\rho)) + \frac{1}{\gamma+1}\rho\hat{K}_f(\rho)\Pi_f'(\rho\hat{K}_f(\rho)) \\ &= R_f(\rho\hat{K}_f(\rho)) \\ &= \hat{K}_f(\rho).\end{aligned}$$

This would lead to later investment, completing the proof of the second part of the proposition.

Proof of Proposition 4

Integrating (2) by parts gives

$$\begin{aligned}R_f(a) &= \frac{\gamma}{\gamma+1}\cdot\frac{1}{r}\left(\pi(a) + a\left(\int_0^1 d\left(\frac{y^{\delta-1}}{\delta-1}\right)\pi'(a/y)\right)\right) \\ &= \frac{\gamma}{\gamma+1}\cdot\frac{1}{r}\left(\pi(a) + a\left(\frac{\pi'(a)}{\delta-1} + \frac{a}{\delta-1}\int_0^1 y^{\delta-3}\pi''(a/y)dy\right)\right)\end{aligned}$$

for all $a > 0$. Now, $\delta > 1$, so that

$$R_f(a) \leq \frac{\gamma}{\gamma+1}\cdot\frac{1}{r}\left(\pi(a) + \frac{a\pi'(a)}{\delta-1}\right).$$

The assumption that $r \leq \eta$ implies that $1/(\delta-1) \leq 1/\gamma$, whence

$$R_f(a) \leq \frac{\gamma}{\gamma+1}\cdot\frac{1}{r}\left(\pi(a) + \frac{a\pi'(a)}{\gamma}\right) = R_b(a).$$

Let $\rho_1 > 0$ be arbitrary, and define $\hat{K}_b^1 = \hat{K}_b(\rho_1)$ to be the investment threshold under the corresponding BL rule. Let

$$\rho_2 = \frac{\rho_1\hat{K}_b^1}{R_f(\rho_1\hat{K}_b^1)}.$$

Since

$$R_f(\rho_1\hat{K}_b^1) \leq R_b(\rho_1\hat{K}_b^1) = \hat{K}_b^1, \tag{4}$$

it follows immediately that $\rho_2 \geq \rho_1$. Furthermore, $R_f(\rho_1\hat{K}_b^1)$ is easily shown to equal the investment threshold under the FL rule with rental rate ρ_2. Therefore, since $\hat{K}_f(\rho)$ is increasing in ρ, we have

$$\hat{K}_f(\rho_1) \leq \hat{K}_f(\rho_2) = R_f(\rho_1\hat{K}_b^1) \leq \hat{K}_b^1 = \hat{K}_b(\rho_1),$$

where we have used (4) to obtain the second inequality in this expression. Since ρ_1 was arbitrary, the proof is complete.

Proof of Proposition 5

The following interpretation of $R_f(a)$ will be useful.

Lemma 6 *Let $\tilde{z}$ be the random variable with support $[1, \infty)$, density function $f(z) = \delta z^{-(\delta+1)}$ and distribution function $F(z) = 1 - z^{-\delta}$. Then*

$$R_f(a) = \frac{\gamma}{\gamma + 1} \cdot \frac{1}{r} \cdot E[\pi(a\tilde{z})].$$

PROOF: The expected value equals

$$E[\pi(a\tilde{z})] = \delta \int_1^\infty z^{-(\delta+1)} \pi(az) dz.$$

The change of variable $z \mapsto 1/y$ leads to

$$E[\pi(a\tilde{z})] = \delta \int_0^1 y^{\delta-1} \pi(a/y) dy.$$

Inspection of equation (1) completes the proof. ■

We start by proving that there exists an increasing function $\delta^*(\gamma)$ such that the BL rule leads to earlier investment if and only if $\delta > \delta^*(\gamma)$.

The investment threshold under the BL rule is $\hat{K}_b = R_b(\hat{a}_b)$, where $\hat{a}_b$ is defined implicitly by $\hat{a}_b = \rho R_b(\hat{a}_b)$. Similarly, the investment threshold under the FL rule is $\hat{K}_f = R_f(\hat{a}_f)$, where $\hat{a}_f$ is defined implicitly by $\hat{a}_f = \rho R_f(\hat{a}_f)$. Therefore, the BL rule leads to earlier investment than the FL rule if and only if

$$\hat{a}_b = \rho \hat{K}_b > \rho \hat{K}_f = \hat{a}_f.$$

Note that $\hat{a}_b$ satisfies

$$(\gamma + 1) a_b = \frac{\rho}{r} \left(\gamma \pi(\hat{a}_b) + \hat{a}_b \pi'(\hat{a}_b) \right). \tag{5}$$

Implicit differentiation with respect to γ, followed by some tedious manipulation, shows that

$$\frac{\partial \hat{a}_b}{\partial \gamma} = \frac{\hat{a}_b \left(1 - \frac{\rho}{r} \pi'(\hat{a}_b)\right)}{\gamma \left((\gamma + 1) \left(1 - \frac{\rho}{r} \pi'(\hat{a}_b)\right) - \frac{\rho}{r} \hat{a}_b \pi''(\hat{a}_b) \right)}.$$

Combining the inequality (3) with equation (5) gives

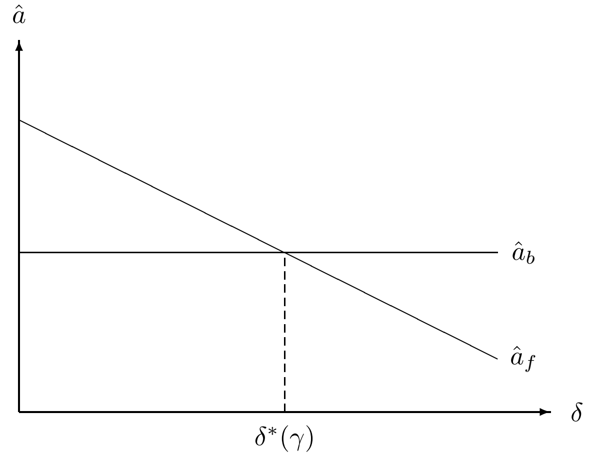
$$(\gamma + 1)\hat{a}_b = \frac{\rho}{r} \left(\gamma \pi(\hat{a}_b) + \hat{a}_b \pi'(\hat{a}_b) \right) > \frac{\rho}{r} (\gamma + 1) \hat{a}_b \pi'(\hat{a}_b),$$

implying that $1 > (\rho/r)\pi'(\hat{a}_b)$. The numerator of $\partial \hat{a}_b / \partial \gamma$ is therefore positive. Furthermore, the concavity of π ensures that the denominator of $\partial \hat{a}_b / \partial \gamma$ is also positive, proving that $\hat{a}_b$ is increasing in γ.

We now turn to $\hat{a}_f$, which satisfies

$$(\gamma + 1) a_f = \gamma \frac{\rho}{r} E[\pi(\hat{a}_f \tilde{z})], \tag{6}$$

Figure 9: Proof of Result 5



where $\tilde{z}$ is the random variable introduced in Lemma 6. Implicit differentiation with respect to γ, followed by some more tedious manipulation, shows that

$$\frac{\partial \hat{a}_f}{\partial \gamma} = \frac{\hat{a}_f}{\gamma\left(\gamma + 1 - \gamma \frac{\rho}{r} E[\tilde{z}\pi'(\hat{a}_f \tilde{z})]\right)}.$$

Combining inequality (3) with equation (6) gives

$$\gamma \frac{\rho}{r} E[\tilde{z}\pi'(\hat{a}_f \tilde{z})] < \gamma \frac{\rho}{r} \frac{E[\pi(\hat{a}_f \tilde{z})]}{\hat{a}_f} = \gamma + 1,$$

so that the denominator of $\partial \hat{a}_f / \partial \gamma$ is positive. Therefore, $\hat{a}_f$ is an increasing function of γ.

It is easily seen that increasing δ shifts the distribution of $\tilde{z}$ to the left, in the sense of first order stochastic dominance. Since $\pi(az)$ is an increasing function of z, this change reduces $E[\pi(a\tilde{z})]$, for any given a. A similar calculation to that above proves that $\hat{a}_f$ is a decreasing function of δ.

Figure 9 plots $\hat{a}_b$ and $\hat{a}_f$ as functions of δ for an arbitrary value of γ. From above, $\hat{a}_b$ is constant and $\hat{a}_f$ is decreasing. The value of $\delta^*(\gamma)$ can be found from the point where the two curves intersect. Suppose that γ and δ are such that $\hat{a}_b$ and $\hat{a}_f$ take some common value $\hat{a}$. Noting that

$$\hat{a}\pi''(\hat{a}) < 0 < \gamma \left(1 - \frac{\rho}{r}\pi'(\hat{a})\right) E[\tilde{z}\pi'(\hat{a}\tilde{z})],$$

it is straightforward to show that

$$\frac{d\hat{a}_b}{d\gamma} < \frac{d\hat{a}_f}{d\gamma}.$$

Returning to Figure 9, where the two curves cross, $\hat{a}_f$ is more sensitive than $\hat{a}_b$ to small changes in γ. Since both curves move up when γ increases, the curve labelled $\hat{a}_f$ moves up further. The point of intersection must move to the right. That is, increasing γ raises δ^*.

Note that

$$\frac{\partial \gamma}{\partial (r-\eta)/\sigma^2} > 0, \quad \frac{\partial \delta}{\partial (r-\eta)/\sigma^2} < 0.$$

Therefore, holding σ^2/r fixed and increasing $(r-\eta)/\sigma^2$ causes γ to rise, and δ^* to rise with it, while δ falls. Thus, when $(r-\eta)/\sigma^2$ is sufficiently large, δ will be less than $\delta^*(\gamma)$, and $\hat{a}_f$ will be greater than $\hat{a}_b$. Thus, there exists a function $N(\cdot)$ such that $\hat{a}_b > \hat{a}_f$ if and only if $(r-\eta)/\sigma^2 < N(\sigma^2/r)$. We define

$$M(\sigma^2/r) = 1 - (\sigma^2/r)N(\sigma^2/r).$$

Then $\hat{a}_b > \hat{a}_f$ if and only if $(r-\eta)/r < (1 - M(\sigma^2/r))$.

Proof of Lemma 5

The proof follows that of Lemma 4, with Π_f replaced by Θ_f everywhere.

Proof of Proposition 7

Consider the constant charging rule with the arbitrary positive access charge $\hat{a}$. Since the functions $R_b(\cdot)$ and $R_c(\cdot)$ are continuous, with $R_b(0) = R_c(0)$ and $R_b(a) > R_c(a)$ for all $a > 0$, it follows that there exists a number $a' \in (0, \hat{a})$ such that $R_b(a') = R_c(\hat{a})$. Since $\theta(\cdot)$ is decreasing, $\theta(a') > \theta(\hat{a})$. The regulator's objective function takes the value

$$\left(\frac{\theta(\hat{a})}{r} - R_c(\hat{a})\right)(R_c(\hat{a}))^{\gamma}$$

under the constant access charge, and the greater value

$$\left(\frac{\theta(a')}{r} - R_b(a')\right)(R_b(a'))^{\gamma}$$

under the BL regime with access charge a'. Clearly, the second case is preferred by the regulator, completing the proof.