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Income Taxation and Progressivity: A Measure of Equitability*

John Creedy† and S. Subramanian‡

Abstract

This paper proposes a real-valued measure of 'equitability of taxation', in the context of progressivity and the income tax. The point of departure is the well-known Kakwani progressivity Index, regarded as a measure of the disproportionality of tax payments. By postulating specifications of 'extreme equitability' and 'extreme inequitability' of a tax system, the paper advances a measure of tax equitability as the normalized area distance of a 'tax concentration curve' from its most inequitable version. The measure is derived and described; a procedure is outlined for the decomposition of differences in the measure across regimes into a 'distribution effect' and a 'tax system' effect; and a criterion is established for asserting 'unambiguously greater equitability' in comparisons across regimes, in terms of a dominance relation akin to the Lorenz quasi-ordering. The measure is illustrated with the help of numerical examples.

JEL Classification: D30, D31, D63, H20, H23, H24

Keywords: Kakwani Index; proportionality; progressivity; Index of equitability; decomposition; T-dominance.

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1 Introduction

The aim of this paper is to introduce a new measure of income tax progressivity, referred to as 'tax equitability', based on considerations of the extent to which the tax has an income equalising effect. As usual, the context is one in which the pre-tax income distribution remains unchanged, and is thus not affected by adverse incentive effects of taxation. Reference is often make to the 'redistributive' effects of taxation, but of course only one form of tax – income taxation – is being considered, and there is no actual redistribution, for example, in the form of social transfer payments.[1] While it has long been recognised that a complete evaluation of the progressivity and distributional effects of taxation needs to allow for all forms of taxes and benefits (for example, there may be a legitimate role for a 'regressive' tax as part of a 'progressive' tax system) the importance of income taxation in developed economies warrants a special focus on its characteristics considered in isolation.[2]

A progressive (inequality reducing) tax requires average tax rates to increase with income. With exclusive focus on positive income taxation, it is clear that progressivity requires some disproportionality in tax payments, that is, a tax schedule having 'rate progression', defined as increasing marginal tax rates.[3] Consider the simple case of a piecewise-linear schedule in which individuals with pre-tax income, x, below a threshold, a, pay no tax, but those with $x \geq a$ pay tax of $T(x) = t(x - a)$. This displays rate progression, as it has marginal rates of 0 and $t > 0$. A strictly proportional income tax (with $a = 0$) necessarily implies equal values of a relative measure of the inequality of pre- and post-tax incomes. This contrasts with the simplest linear tax-and-transfer system in which there is a universal basic income and a proportional income tax which, in the absence of incentive effects, can achieve complete equality in post-tax incomes (unlike an income tax alone, except of course in the trivial case of a proportional tax imposed at 100 per cent of income).

The point of departure of the present analysis is the now-standard approach to measuring income tax progressivity, following the seminal work of Kakwani (1977), whose index, making use of the well-known tax concentration curve, measures the disproportionality of tax payments. Importantly, the measure can be directly related to the extent of redistribution, where the latter is taken to be the difference between Gini inequality measures of pre-tax and post-tax income. Kakwani's measure is examined

[1] It also excludes public expenditure which may benefit some individuals more than others.

[2] An example of another tax that is often considered in isolation is the Value Added Tax, or other expenditure taxes, despite the additional complication that expenditure is financed from post-income-tax income, and savings.

[3] In general, in a structure having benefits as well as taxes, rate progression is not necessary for progressivity.

in Section 2, where several problems are highlighted. These problems – namely the infeasibility of the extreme tax distributions involved in constructing the measure, and its failure to allow for changes in disproportionality in the simple tax function mentioned above – motivate the new measure introduced in Section 3. By postulating specifications of 'extreme equitability' and 'extreme inequitability' of a tax system, the section advances a measure of 'tax equitability' as the normalized area distance of a tax concentration curve from its most inequitable version. The properties of the new measure are illustrated in Section 4, and brief conclusions are in Section 5.

2 The Kakwani Measure

This section defines the Kakwani measure, in Subsection 2.1, and then discusses two difficulties with the measure, in Subsection 2.2.

2.1 Definition of K

The basic idea behind the Kakwani measure, K, is that a tax-concentration curve can be produced, for which the cumulative proportion of total tax revenue is plotted against the corresponding proportion of people, where taxpayers are ranked in ascending order according to their pre-tax incomes.[4] The curve is referred to as a concentration curve rather than a Lorenz curve of taxation, because the ranking by tax paid does not necessarily correspond to the ranking by pre-tax income, in a system in which there are numerous non-income factors involved. Even in the simple tax schedule mentioned in Section 1, the threshold, a, may differ between taxpayers if there are various deductions and allowances, which may be based on family characteristics.

Kakwani (1977) proposed that a concentration measure, C_T, can be obtained just as a Gini inequality measure, in terms of the normalised area between the tax concentration curve and the diagonal 'line of equality' along which everyone pays the same absolute tax. For a progressive system, (relative) tax payments must be more unequal than pre-tax incomes: they have to be disproportional. Hence, the tax concentration curve must be 'outside' the Lorenz curve of pre-tax income, and $C_T > G_x$.[5] An example of hypothetical Lorenz and tax concentration curves is shown in Figure 1.

The Kakwani measure of disproportionality (the extent of the income-equalising

[4] For an introduction to the measurement of progressivity, see Creedy (2000).

[5] It is possible (in the face of strong reranking and thus large non-income differences among the poor) for the tax concentration curve to lie above the diagonal in early ranges. However, this does not seem to arise in practical cases.

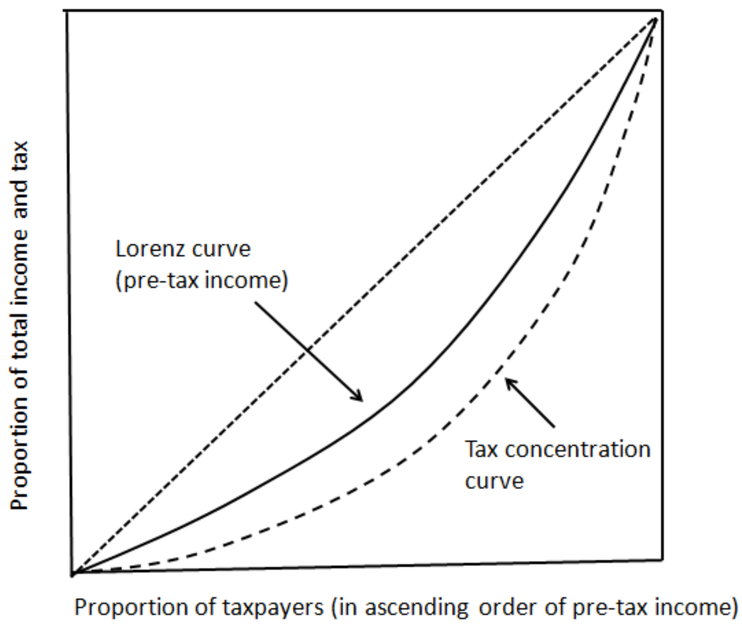

Figure 1: A Lorenz Curve of Pre-Tax Income and Associated Tax Concentration Curve

deviation from a strictly proportional tax) is thus:

$$K = C_T - G_x \tag{1}$$

Importantly, the value of K depends both on the pre-tax distribution and the total revenue collected by the tax, $R = \sum_{i=1}^{n} T(x_i)$. This captures the fact that an income tax that has high marginal rates over certain income ranges cannot achieve progressivity if there are few incomes in those ranges, and disproportionality can have only a small effect if very little revenue is collected.

In the case where there is no re-ranking when moving from pre- to post-tax incomes, then of course $C_T = G_T$: 'tax concentration' is the same as a measure of the inequality of tax payments. In this special case, the redistributive effect of the income tax, $G_x - G_y$, where $y = x - T(x)$ is post-tax income, can be expressed as:[6]

$$G_x - G_y = K\left(\frac{\tau}{1-\tau}\right) \tag{2}$$

Here τ is the aggregate tax rate: for a population of n taxpayers, $\tau = R/n\bar{x}$, where $\bar{x}$ is arithmetic mean income. In the case where there is some re-ranking, (2) must be modified by subtracting a measure of reranking.[7] Reranking is not treated separately here, since it has no effect on the measures considered below, which focus on the concentration curve.

[6] See Kakwani (1984).

[7] This is the Atkinson (1980)–Plotnick (1981) reranking measure, given by the difference between

2.2 Some Problems

It is worth pausing to consider the assumptions implicit in the use of a Gini-type measure of inequality applied to the tax concentration curve: the term 'concentration' is perhps slightly awkward here, since the Gini-based concentration measure is actually a measure of 'dispersion', but with individuals ranked by pre-tax incomes. This carries with it, as with the standard Gini and the Lorenz curve, the idea that two hypothetical extremes, of complete equality and complete inequality, can be defined. 'Complete equality' in the concentration curve diagram corresponds to the leading diagonal: all individuals pay the same amount of tax. 'Complete inequality' of tax payments corresponds to the case where only the richest person attracts the tax revenue, so that (for large n) the concentration curve follows the base and right hand sides of the 'box' diagram.

These definitions of the two extremes, while they are clear in the case of income inequality and the Lorenz curve, are nevertheless problematic in the present taxation context. Specifically, for the postulated extremes to be compatible with the total tax raised, strong restrictions need to be placed on the feasible range of R. The 'maximally equitable' extreme requires the entire tax burden to fall on the richest individual, while ensuring that this person's post-tax income does not slip below the incomes of those less rich. The 'maximally inequitable' extreme requires that the poorest individual can afford an equal share of the aggregate tax burden. Together, these 'extremal conditions' impose the following constraint on the permissible size of tax revenue raised: $R \leq \min(nx_1, x_n - x_{n-1})$. This is a requirement of such extreme restrictiveness as to confine the applicability of the framework of analysis to rather unrealistic contexts. The problem has not gone unnoticed in the literature: it has been explicitly discussed in Mantovani (2017) and Mantovani *et al.* (2018).

From one point of view these aspects of the definition of the tax concentration measure would not matter, in defining the measure of progressivity, K, as reflecting the disproportionality of tax which leads to redistribution. Recognising that a strictly proportional tax (with no reranking) has no effect on relative inequality, K could be directly defined as the normalised area distance of the tax concentration curve from the Lorenz curve of pre-tax income, since these would coincide under the proportional tax. Evaluation of this area would be equivalent to $C_T - G_x$, though the use of C_T is conveniently bypassed.

the Gini measure of post-tax income and the concentration measure of post-tax income (that is, where individuals are ranked according to pre-tax incomes). Reranking may be said to 'frustrate' the progressive aim of the tax to some extent. Aronson and Lambert (1994) showed how the redistributive effect can be further decomposed into components of 'vertical' and 'horizontal' equity.

However, consider again the simple tax function described above for which individuals with pre-tax income, x, below a threshold, a, pay no tax, but those with $x \geq a$ pay tax of $T(x) = t(x-a)$. This tax structure clearly does not have $T(x)$ proportional to x. Consider the resulting tax concentration curve, showing the proportion of total tax corresponding to a given proportion of taxpayers. First, following Creedy (1996), total tax revenue can be written as:

$$R = t\bar{x}\left[(1 - F_1(a)) - \left(\frac{a}{\bar{x}}\right)(1 - F(a))\right] \tag{3}$$

where $\bar{x}$ is the arithmetic mean of x, and $F(x)$ and $F_1(x)$ denote the distribution function, and first moment distribution function, respectively of x. The term within square brackets in (3) depends only on the threshold, a, and the form of the fixed distribution of x. The proportion of total tax revenue contributed by a taxpayer (that is, someone for whom $x > a$) is thus $t(x-a)$ divided by R. It is immediately obvious that t cancels from this ratio. Hence a change in the tax rate, t, has no effect on the concentration curve, and hence the value of K. Yet such a tax structure is non-proportional: there are two marginal rates (0 and t) and the average tax rate increases, with the rate of increase depending on t and a. Hence the problem with K is more than simply a question of defining the concentration ratio, which, as seen above, could be bypassed. It fails, in the case of the simple tax structure discussed here, to record a change in tax disproportionality, which it is designed to measure. This suggests a need for an alternative measure of the distribution of tax payments, and their possible equalising effect.

3 A New Measure

The recognition that the 'extreme' distributions involved in the definition of the concentration measure, C_T, are not realistic alternatives (even with a completely omnipotent tax authority and no adverse incentive effects), along with the fact that the resulting value of $K = C_T - G_x$ can fail to capture a change in tax disproportionality, motivates the present section. Subsection 3.1 proposes a measure of the 'equitability' of taxation, reflecting the extent of progressivity (viewed in terms of the redistributive effect of the tax). Subsection 3.2 then demonstrates the conditions under which one tax structure could be said unambiguously to reflect more equitability than another structure. Subsection 3.3 briefly explains how changes in the measure can be decomposed into separate changes contributed by income distribution and tax structure changes.

3.1 A Measure of Equitability

The point of departure of the present contribution is that it is possible to define the extreme tax distribution cases in a different and more realistic way. For clarity, terminology is restricted to the progressivity (inequality reducing) characteristics of the tax extremes considered. As mentioned above, the equal-tax case is not feasible, as it presupposes that the poorest people can match the richest in absolute tax payments, while raising sufficient revenue. When considering the least progressive (most inequality-increasing) case, an alternative extreme is such that the burden of taxation falls most heavily on the poorest people, and the highest incomes may even escape tax altogether. The number of poor taxpayers affected depends on their incomes as well as the revenue required. The hypothetical tax concentration curve associated with this extreme is thus likely to be above the leading diagonal for much of its range. In contrast, the other – most income-equalising – extreme is not one in which only the highest income pays tax, but one that involves a 'levelling down' of top incomes. All those above a threshold income (determined by a combination of the income distribution and total revenue required) have the 'excess' income taxed at 100 per cent.[8]

First, it is useful to define the tax concentration curve explicitly. For incomes, x_i, for $i = 1, ..., n$, arranged in ascending order, a proportion, $P_i = i/n$, of taxpyers is responsible for paying a proportion of total tax, equal to $R_i = (1/R) \sum_{j=1}^{i} T(x_i)$. Let $P_0 = R_0 = 0$. The standard tax concentration curve plots the points:

$$(P_0, R_0), (P_1, R_1), ..., (P_j, R_j), ..., (P_n, R_n) \tag{4}$$

in the unit square.

Consider the most inequitable, or least equalising, tax schedule in terms of its effect on net incomes. In this case, the poorest individual (the person for whom $i = 1$) pays tax, say $T^*(x_1)$, equal to either all income, x_1, or the entire tax revenue required, R, whichever is the smallest. Hence:

$$T^*(x_1) = \min(x_1, R) \tag{5}$$

Of course, it is most likely that $x_1 < R$. Moving gradually up the distribution, $T^*(x_i)$, for $i = 2, ..., n$, is given by:

$$T^*(x_i) = \min\left(x_i, R - \sum_{j=1}^{i-1} T^*(x_j)\right) \tag{6}$$

[8] Indeed, the situation in which only the richest person pays tax could reduce that person's income to such a level that overall inequality of net income actually increases.

This applies so long as $T^*(x_i) \geq 0$, otherwise individual i pays no tax, and $T^*(x_i) = 0$. This extreme schedule of tax payments replaces the leading diagonal used in producing the Kakwani C_T measure. It is likely to be substantially above the diagonal except perhaps for those in the very lowest-income ranges.

Consider next a feasible most-equalising case. This reduces the higest income down to the level of the next-richest person, and is achieved with a marginal rate of 100 per cent applied to income in excess of the second-richest person. This is unlikely to raise sufficient revenue, and so a process continues of equalising the top incomes above a particular threshold, determined by total revenue required and the distribution of higher incomes. Thus, suppose k is the largest integer such that:

$$R = \sum_{i=k}^{n} (x_i - x_k) \tag{7}$$

Then, for $i = 1, ..., k-1$, tax is $\widehat{T}(x_i) = 0$. For $i = k, ..., n$, tax is given by:

$$\widehat{T}(x_i) = x_i - x_k \tag{8}$$

For large numbers, the condition in (7) can be met sufficiently closely. However, with small numbers, some further adjustment to the effective threshold, x_k, is likely to be required. This extreme is equivalent to a tax system with a single threshold of x_k above which the marginal tax rate is 100 per cent. The tax function, $\widehat{T}(x_i)$, is in fact the one postulated in Mantovani (2017) and Mantovani *et al.* (2018) in the context of analysis of the maximum value of K.[9] It was previously proposed by Jayaraj and Subramanian (2010) in the context of poverty eradication through redistributive taxation.

Using the resulting $\widehat{T}(x_i)$ and $T^*(x_i)$ values, along with the P_is as defined above, it is possible to construct two tax concentration curves, representing the most- and least-equalising cases respectively. Hypothetical examples of the two curves, along with the actual tax concentration curve, are shown in Figure 2. The relevant curves are labelled $\mathrm{C_B}$ and $\mathrm{C_W}$, indicating – from a redistributive point of view – the 'best' and 'worst' extremes.

The standard tax concentration measure of the inequality of tax payments, based on Gini-type comparisons, is equal to twice the area between the tax conentration curve and the diagonal line: it measures a (normalised) 'distance' of the curve from the case where all individuals pay the same tax. Let the area beneath the tax concentration curve be denoted A_C. Then by analogy with the Gini measure, $C_T = 1 - 2A_C$. The above discussion suggests an alternative measure, say E_T, that instead captures the *equitability* of income taxation, and is defined as follows. Denote the areas *underneath*

[9] They qualify the result that the maximum value K can take is $1 - G_x$.

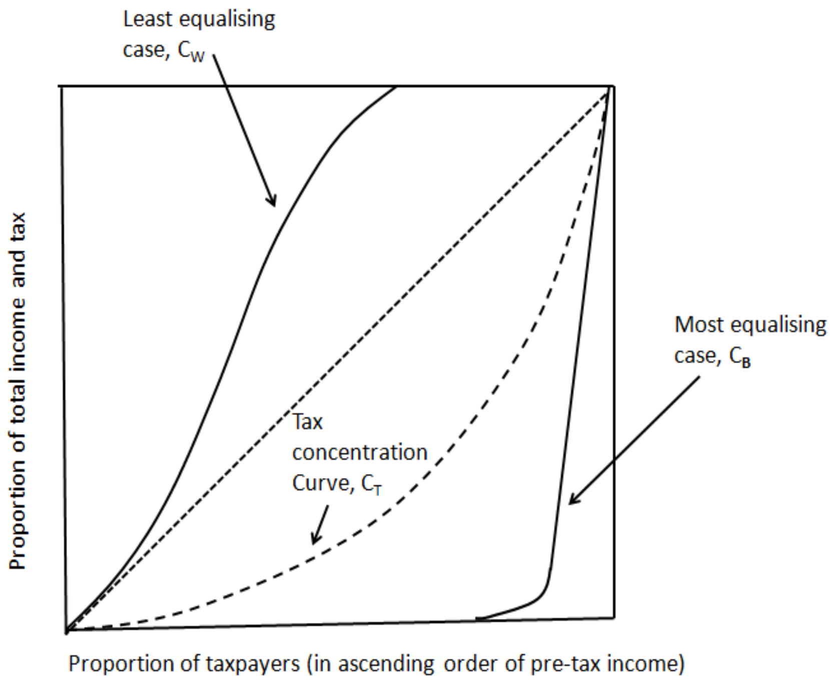

Figure 2: Hypothetical Extreme Tax Concentration Curves

the C_W and C_B curves respectively as A_W and A_B. Then E_T is the area between the tax concentration curve and the C_W curve, normalised by the area contained by the two extreme cases. Importantly, this is a normalised measure of the 'distance' (expressed as an area) between the tax concentration curve and the most-equalising (in terms of incomes) extreme.[10] Hence:

$$E_T = \frac{A_W - A_C}{A_W - A_B} \tag{9}$$

The various areas needed for the computation of (9) can of course be obtained using the well-known trapezoidal rule.

One desirable property of the measure corresponds to a type of Dalton-Pigou 'principle of transfers'. For a given distribution of pre-tax income, and a fixed value of total revenue, a rank-preserving transfer of tax liability from person i to person j, where $x_i < x_j$, increases the equitability of tax, E_T. The tranfer moves the tax concentration curve closer to C_B over part of its range, but of course the two curves C_W and C_B remain unchanged.

The suggestion here is that the equitability measure in (9) has a more meaningful interpretation than the standard tax concentration measure, C_T, on which the Kakwani tax progressivity measure is based. In addition, unlike K, E_T is sensitive to changes

[10]In the Kakwani concentration measure, both A_W and $A_W - A_B$ are assumed to be equal to 0.5.

in the tax structure in cases where K does not reflect disproportionality changes. The relationship between the two measures can be seen as follows. Rearranging (9), the area below the tax concentration curve is:

$$A_C = (1 - E_T)\, A_W + E_T A_B \tag{10}$$

and A_C is a weighted average of the areas below the 'worst' and 'best' tax concentration curves (from the point of view of achieving income redistribution), with weights depending in the tax equitability measure. Using $C_T = 1 - 2A_C$, the Kakwani measure can be written as:

$$K = 1 - 2\left\{(1 - E_T)\, A_W + E_T A_B\right\} - G_x \tag{11}$$

Hence, using (2) the redistributive effect (in the absence of reranking) is given, in terms of E_T and its component areas, as:

$$G_x - G_y = \left(\frac{\tau}{1-\tau}\right)\left[1 - 2\left\{(1 - E_T)\, A_W + E_T A_B\right\} - G_x\right] \tag{12}$$

3.2 Comparing Tax Equitability

This subsection explores the possibility of obtaining a condition under which one tax structure is unambiguously more inequality-reducing than another.[11] First define a 'regime', X, as a pair consisting of the pre-tax income distribution, $\mathbf{x} = (x_1, ..., x_n)$, and the accompanying tax schedule, so that $X \equiv [\mathbf{x}, T(x_i)]$.

The concentration curve plots the points listed in (4) above. To avoid confusion, when comparing alternative structures, it is convenient to rewrite these as:

$$(P_0, Q_0), (P_1, Q_1), ..., (P_n, Q_n) \tag{13}$$

where incomes are arranged in ascending order, and the proportion, $P_i = i/n$, of taxpayers is responsible for paying a proportion, Q_i, of total revenue, with $P_0 = Q_0 = 0$. The Q_i may conveniently be written as:

$$Q_i = Q_i\left([\mathbf{x}, T(x_i)]\right) = \frac{1}{R}\sum_{j=1}^{i} T(x_i) \tag{14}$$

The tax concentration curves corresponding to the most and least equitable tax regimes were denoted above by $\mathrm{C_B}$ and $\mathrm{C_W}$ respectively. Hence, the former is obtained by plotting the points $\left(P_0, \hat{Q}_0\right), \left(P_1, \hat{Q}_1\right), ..., \left(P_n, \hat{Q}_n\right)$, with:

$$\hat{Q}_i = \hat{Q}_i\left(\left[\mathbf{x}, \hat{T}(x_i)\right]\right) = \frac{1}{R}\sum_{j=1}^{i} \hat{T}(x_i) \tag{15}$$

[11] Such a condition is akin to the familiar Lorenz-dominance relation between income distributions.

And similarly for $\mathrm{C_B}$, where:

$$Q_i^* = Q_i^*\left([\mathbf{x}, T^*(x_i)]\right) = \frac{1}{R}\sum_{j=1}^{i} T^*(x_i) \tag{16}$$

The definition of E_T in (9) involves areas under the three concentration curves. However, this can be converted to comparisons of two areas, as follows. Define a new curve, D^*, obtained by plotting $(P_0, D_0^*), (P_1, D_1^*), ..., (P_n, D_n^*)$ where $D_i^* = Q_i^* - \hat{Q}_i$. Similarly, define the new curve, $\hat{\mathrm{D}}$, as plotting $\left(P_0, \hat{D}_0\right), \left(P_1, \hat{D}_1\right), ..., \left(P_n, \hat{D}_n\right)$, with $\hat{D}_i = Q_i^* - Q_i$. Both the D^* and $\hat{\mathrm{D}}$ curves have inverted U-shapes, commencing at $(0,0)$ and terminating at $(1,0)$. Denoting the area under each of these curves by A_{D^*} and $A_{\hat{D}}$ respectively, E_T can be rewritten as:

$$E_T = \frac{A_{\hat{D}}}{A_{D^*}} \tag{17}$$

Given any two tax regimes X and X', X can be said to D^*-dominate X' weakly, written $X \geq_{D^*} X'$, if and only if the D^* curve for X lies nowhere outside the D^* curve for X'; furthermore, regime X is said to D^*-dominate X' strictly, written $X >_{D^*} X'$, if and only if the D^* curve for X lies somewhere inside and nowhere outside the D^* curve for X'. The dominance relations, $X \geq_{\hat{D}} X'$ and $X >_{\hat{D}^*} X'$ can be analogously defined.

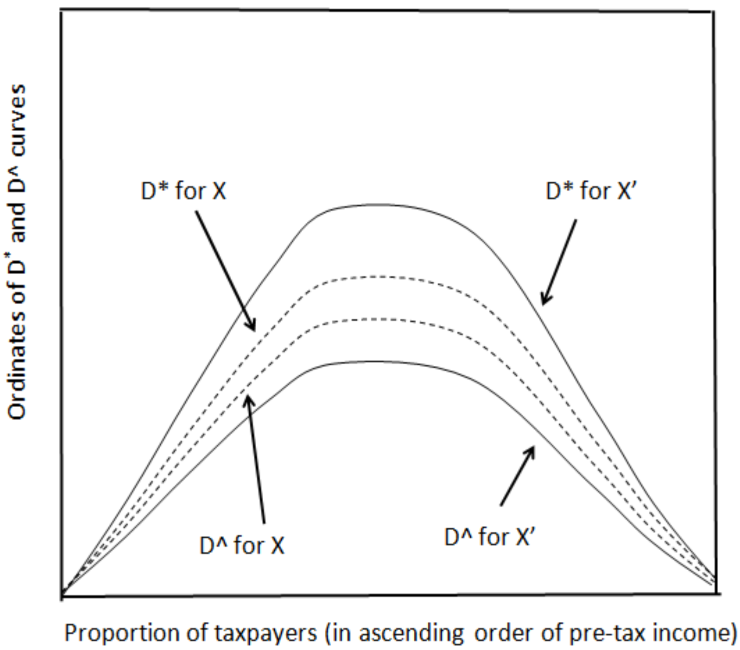

Figure 3: Tax Equitability Dominance

Given two regimes X and X', taxation under X is unambiguously more equitable than taxation under regime X', written $X >_T X'$, whenever (i) $X \geq_{D^*} X'$ and $X' >_{\hat{D}} X$, or (ii) $X >_{D^*} X'$ and $X' \geq_{\hat{D}} X$, or (iii) both $X >_{D^*} X'$ and $X' >_{\hat{D}^*} X$. Case (iii) is illustrated in Figure 3. As the figure illustrates, whenever $X >_T X'$, the area under the $\hat{\mathrm{D}}$ curve for X is no smaller than that under the $\hat{\mathrm{D}}$ curve for X', while the area under the D* curve for X' is no smaller than that under the D* curve for X, with the relevant area being, in at least one case, strictly more. And since the E_T measure is just the ratio of the area under the $\hat{\mathrm{D}}$ curve to the area under the D* curve, it follows that whenever $X >_T X'$ holds, E_T should be larger for X than for X', though the converse need not hold.

3.3 Decomposing Changes in E_T

It has been stressed that the progressivity of taxation, here measured by the tax-equitability measure, E_T, depends on both the tax structure and the distribution of pre-tax incomes. Hence, it is possible to have a tax structure which appears at first sight to be less equalising than another, but can have the same redistributive effect, because of the differing nature of the pre-tax income distributions. The present subsection shows how changes in the tax equitability measure can be decomposed into separate tax and income distribution contributions, following the general approach proposed by Shorrocks (2013).

It is convenient to write E_T as a function of the income distribution, X, and the tax structure, T, so that $E_T = E(X, T)$. Changes between period 0 and period 1 can be decomposed in two ways, as follows:

$$E(X_1, T_1) - E(X_0, T_0) = [E(X_1, T_1) - E(X_1, T_0)] + [E(X_1, T_0) - E(X_0, T_0)] \quad (18)$$

$$E(X_1, T_1) - E(X_0, T_0) = [E(X_0, T_1) - E(X_0, T_0)] + [E(X_1, T_1) - E(X_0, T_1)] \quad (19)$$

In each case, the first term in square brackets measures the tax-structure effect, for a given income distribution. The second term in square brackets measure the income-distribution effect, for a given tax structure. As there is no reason to prefer one decomposition over the other, arithmetic mean values can be used. Of course, these can easily be converted into percentage contributions. Clearly, the same kind of decomposition can be applied to Gini inequality measures and the Kakwani measure.

4 A Numerical Illustration

In order to clarify the equitability concept and its properties, this subsection presents a small numerical example based on a hypothetical population consisting of just ten

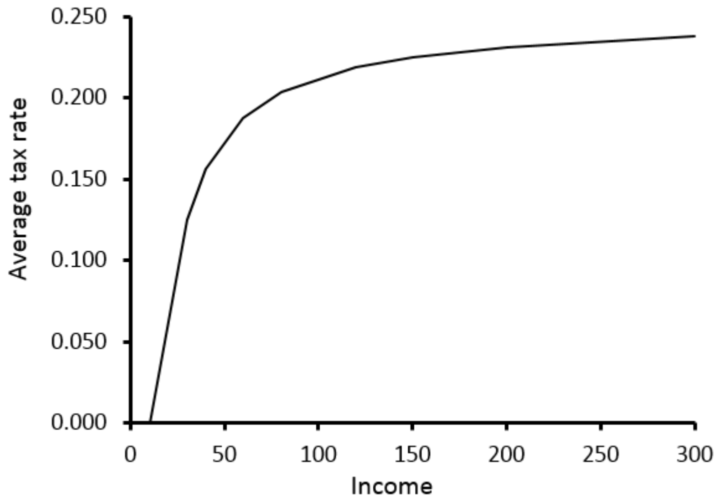

Figure 4: Average Tax Rate: a = 15; t = 0.25

taxpayers. The various concentration curves are illustrated in Subsection 4.1. Examples of the effects of changes in the tax structure are given in Subsection 4.2, using the simple tax structure discussed above, involving the two rates 0 and t and a tax-free income threshold of a. Decompositions are considered in Subsection 4.3, for the E_T measure along with G_y and K.

4.1 Examples of Concentration Curves

Suppose $n = 10$ and the pre-tax incomes, arranged in ascending order, are:

$$[10, 20, 30, 40, 60, 80, 100, 150, 200, 300] \tag{20}$$

Suppose the tax schedule takes the simple form for which individuals with pre-tax income, x, below a threshold, a, pay no tax, but those with $x \geq a$ pay tax of $T(x) = t(x - a)$. Furthermore, let $a = 15$ and $t = 0.25$. This produces an increasing average tax rate, above the tax-free threshold, as shown in Figure 4: by definition the structure is therefore progressive.

The various concentration curves are shown in Figure 5. In this example, it can be found that the effective tax threshold for the most-equalising case (above which the marginal rate is 100 per cent) is equal to 144.58, which is slightly below the income of the third-richest person. The associated D* and $\hat{\text{D}}$ curves are shown in Figure 6. The resulting summary measures are presented in Table 1.

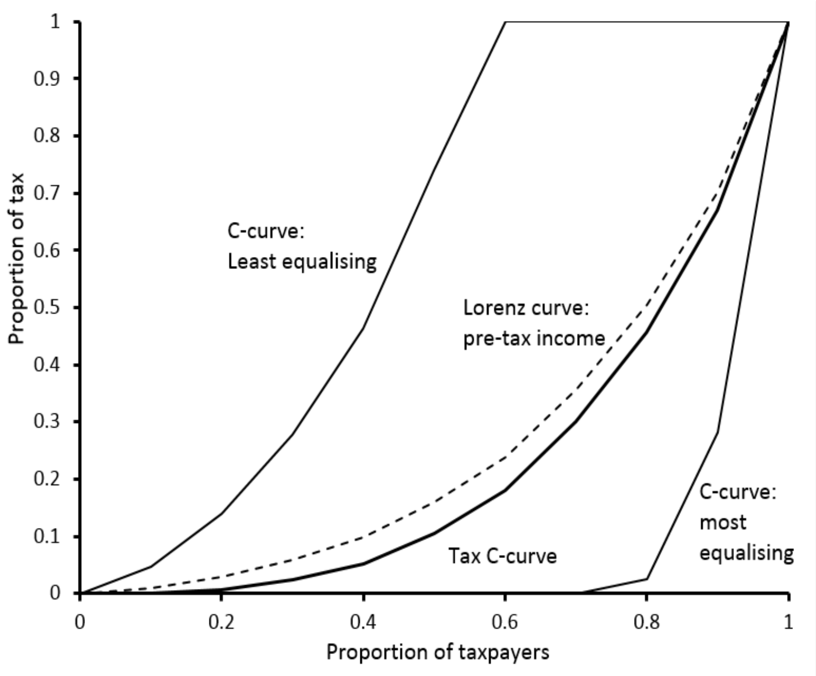

Figure 5: Lorenz and Tax Concentration Curves

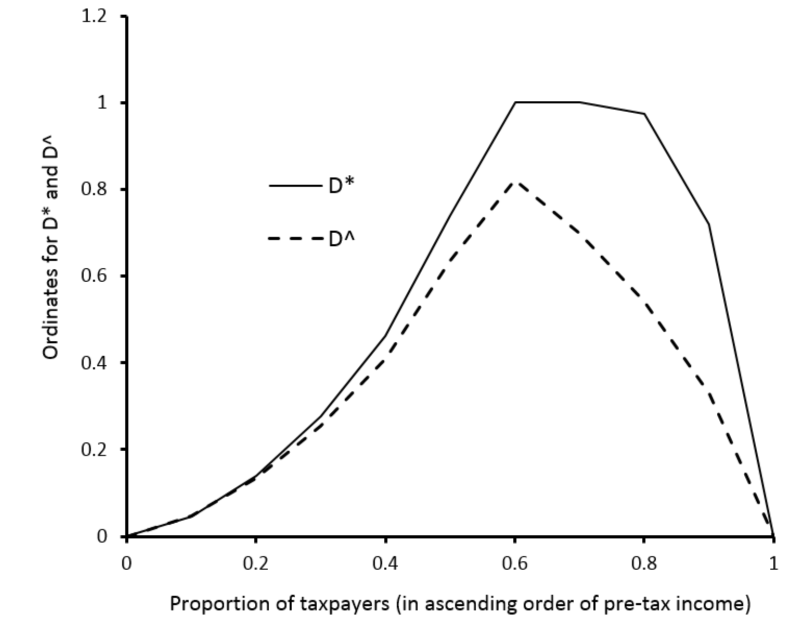

Figure 6: The D* and $\hat{\mathrm{D}}$ Curves

Table 1: Numerical Example: Summary Measures

Pre-tax Gini, G_x	0.4683
Post-tax Gini, G_y	0.4483
Tax concentration, C_T	0.5416
Redistributive effect, $G_x - G_y$	0.020
Kakwani progressivity, $K = C_T - G_x$	0.0733
Aggregate tax rate, τ	0.2141
Area below curve:	
Tax concentration curve, A_C	0.2292
Least progressive case, A_W	0.6165
Most progressive case, A_B	0.0500
Equitability, $E_T = (A_W - A_C) / (A_W - A_B)$	0.7228

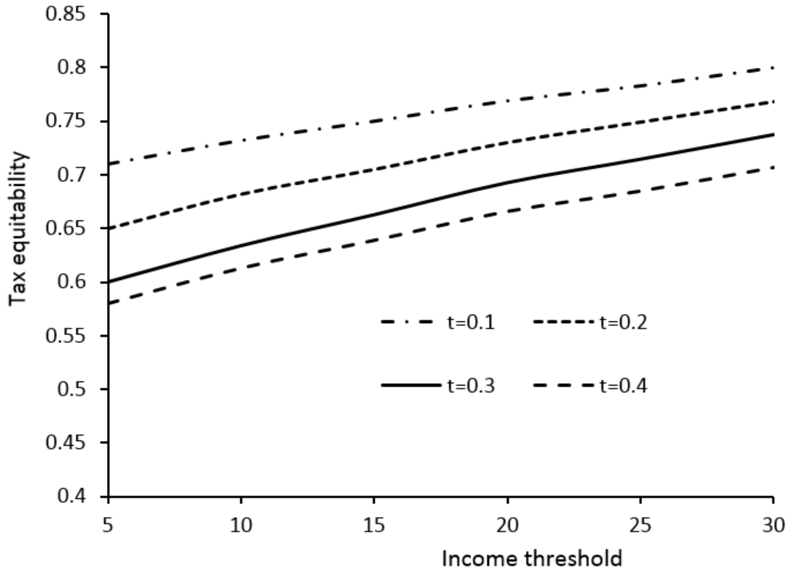

Figure 7: Variations in E_T as a and t Vary

4.2 Variations in the Tax Structure

It is of interest to examine how equitability varies as the tax structure is varied. Figure 7 first shows how E_T varies, for variations in a and t. Next, consider revenue neutral changes in the structure. First, the variation in the overall average tax rate, τ, for variations in a and t, is shown in Figure 8. This information can be used to obtain the value of t, for a given a, which produce a specified overall average tax rate. One feature of revenue-neutral comparisons is that the two curves, $\mathrm{C_B}$ and $\mathrm{C_W}$, remain unchanged, since these depend only on the pre-tax income distribution and total tax revenue. Suppose it is required to keep τ constant at 0.25. The resulting values of E_T, are shown in Figure 9, plotted against the tax-free threshold, a. The required value of t, for each a value on the horizontal axis, to maintain constant τ is also plotted, along with the redistributive effect, $G_x - G_y$.

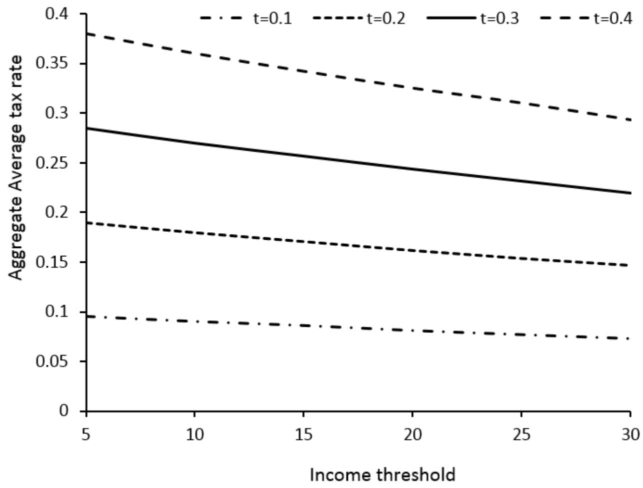

Figure 8: Variations in τ as a and t Vary

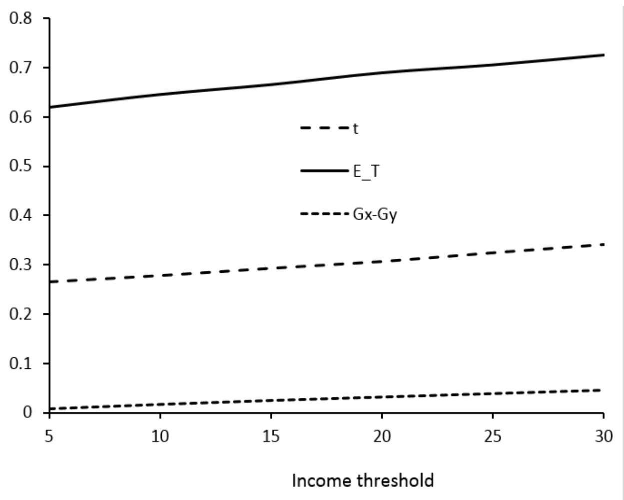

Figure 9: Revenue Neutral Comparisons

4.3 Some Decompositions

Suppose the income distribution at time, 0, is that given in (20), and the tax structure has $a = 10$ with $t = 0.25$. In period 1 the income distribution is the same as in (20) except that the top three incomes are instead lower, at 140, 160 and 190. The tax structure in period 1 has $a = 20$ and $t = 0.2$. Thus there is less inequality of pre-tax incomes in the second period, and a change in the tax structure has opposing effects, since the threshold is higher, but the marginal rate, t, is lower. It is found that the E_T measure of tax equitability rises by 9.64 per cent from period 0 to period 1, while the Gini inequality measure of post-tax income falls by 15.64 per cent. In contrast, the Kakwani measure, K, increases substantially, by a massive 82.35 per cent. This arises because, as shown above, K is not (with this simple tax structure) sensitive to changes in t, which in this case operate to reduce progressivity.

The results of applying the decomposition method of Subsection 3.3 are reported in Table 2. It can be seen that the change in the tax structure makes a relatively small contribution to the reduction in the Gini inequality of post-tax incomes, while making a larger contribution to the (proportionately large) change in K. In this case, the changes in the tax structure and income distribution have opposing effects on K, but the tax structure component dominates. For the more moderate change in E_T, reflecting the opposing effects of changes in a and t, the contribution of the income distribution change is important, at 34.5 per cent, but on balance the percentage tax contribution is almost double that of the income distribution.

Table 2: Percentage Tax and Income Distribution Components

			Percentage components	
Change			Tax structure	Income distribution
$E_{T,1} - E_{T,0}$	=	0.680	65.5	34.5
$G_{y,1} - G_{y,0}$	=	−0.071	5.3	94.7
$K_1 - K_0$	=	0.044	107.8	−7.2

5 Conclusions

This paper has proposed a new measure of income tax progressivity, referred to as a tax equitability measure. The emphasis is on the contribution of the tax structure to the reduction in income inequality when moving from pre-tax to post-tax incomes. The new measure is expressed as the normalised 'distance area' of an appropriately defined tax concentration curve from its most inequitable version. One motivation for the analysis is the finding that the existing well-known measure, the Kakwani measure of disproportionality of tax payments, can in some circumstances fail to recognise relevant changes in the tax structure. Rectification of this deficiency requires an appropriate specification of the extremes of maximum and minimum equitability of a tax system, involving the implementation of relevant 'lexical' formulae at either extreme. For empirical applications involving the comparison of equitability in alternative regimes, the data requirements for estimating the equitability measure are no more demanding than those for estimating the Kakwani measure.

It was shown that the measure satisfies a number of desirable properties, such as an equivalent in the tax context of the Pigou-Dalton principle of transfers. Furthermore, conditions for 'dominance' relationships were established, for judging equitability under one regime to be unambiguously greater than in another regime, akin to those for Lorenz dominance in the income distribution context. Regime-specific differences in the measure can also be decomposed into an effect that is ascribable to differences in the pre-tax income distributions, and one to differences in the tax systems under the regimes in question. Numerical illustrations were provided of the relevant relationships and measures.

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