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by

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ABSTRACT

The Allingham-Sandmo (1972) model of tax evasion in which compliance depends on the perceived probability of detection, the tax rate and the penalty for evasion has been extensively analysed but in the context where detected evasion is assumed to be fully enforced. This paper adapts the A-S model to examine the consequences of partial enforcement of evaded tax. Specifically it models the case where evasion/avoidance take the form of *late payment* of tax subject to penalties, but where these cannot be fully enforced. It then explores how reduced penalty incentives for tax debtors, and penalty rate misperceptions, affect non-compliance decisions.

1. Introduction

The so-called 'standard model' of tax compliance, or evasion, due to Allingham and Sandmo (AS, 1972) has been subjected to widespread empirical testing and challenge – the latter, mainly with respect to its apparently counter-intuitive prediction that evasion is decreasing in the tax rate. Less controversially, the AS model also predicts that compliance increases in the 'fine rate' – levied as a fraction of the tax evaded. However, as Slemrod (2007, p.38) noted: "*there has been no compelling empirical evidence addressing how noncompliance is affected by the penalty for detected evasion, as distinct from the probability that a given act of noncompliance will be subject to punishment*". Additionally, the literature has given little consideration to how the predictions of the AS model are affected where the evaded income can be identified but the associated tax and penalties cannot be fully enforced. Related papers with somewhat different objectives include Slemrod et al. (1995), Skov (2013), and Hallsworth et al. (2014).[1]

This paper uses the AS model to explore 'imperfect enforcement' of evaded tax and penalties in the context of late payment of tax liabilities. Late payment of tax may be best thought of as 'avoidance' rather than evasion (since the tax is not generally hidden illegally from the revenue authority), but it has a particular advantage in this context. That is, since the outstanding tax and penalties are generally acknowledged by both the taxpayer and the revenue authority, the 'probability of detection' in the AS model can reasonably be treated as unity in this case, allowing us to focus on the 'probability of enforcement'. The value of outstanding tax and penalties may, or may not, be agreed between the taxpayer and tax authority, but in either case the tax debt may be thought of as avoidance which the tax authority has a probability of enforcing (collecting) that is less than one.

To illustrate the model's properties in this context we consider the case of the New Zealand late payment penalty regime (for goods & service tax, GST). An advantage of the New Zealand example is that penalty rates are relatively high – an effective rate around 28% – providing a substantial incentive to either pay on time or avoid payment altogether. Since the penalty system is also relatively complex, involving several elements commonly found in other countries, taxpayers' responses may be conditioned by the extent of their knowledge or perceptions of the system. We therefore examine how penalty misperceptions can be expected to affect behaviour.

The remainder of the paper is organised as follows. Section 2 first summarises the key aspects of the AS model in sub-section 2.1. Following some discussion of late payment penalty regimes in 2.2 (using New Zealand's case for illustration), sub-section 2.3 considers how the AS model can be adapted to the case of late payment where penalties are payable but not fully enforceable. This sub-section also considers possible effects on compliance when penalty rates are misperceived and

[1] In a somewhat different context, Slemrod et al. (1995) analysed why some US taxpayers file late or early despite both involving financial losses for many such filers. Their model develops what they call the 'stochastic opportunity cost' of filing – namely the opportunity cost of the time involved in filing which, they argue, can be volatile from day to day and can be treated as random draws from a distribution. More recently, Skov (2013) examines the response of Danish taxpayers' timing decision on payment of taxes owed to the imposition of penalties. Hallsworth et al. (2014) examine empirically how far UK income taxpayers' delay in paying their taxes is consistent with a 'social norm' based explanation, as opposed to a 'crime and punishment' model.

when reduced penalties are subsequently offered to tax debtors. Section 3 then examines some tax debt and late payment data relevant to the model's 'probability of enforcement' parameter, and section 4 draws some conclusions.

2. Modelling Taxpayer Choices

As with the decision to evade tax in the standard AS model, in the case of late payments taxpayers face a double decision: whether to delay payment and hence incur debt and, subject to positive debt being optimal, how much debt to incur. Indeed there is a direct comparison with the AS model if the decision to incur debt is treated as analogous to a decision to evade tax – in this case time-related evasion via delaying payment of period t tax liability till at least period $t+1$.

In this context the term 'avoidance', rather than 'evasion', may be more appropriate since, in general, the taxpayer is not illegally hiding the debt from tax authority, even if the *value* of the debt outstanding may not be agreed by both parties. In addition, though in some sense the avoidance has already been detected, where the taxpayer perceives a probability of permanently avoiding all or part of the debt (e.g. via a write-off), this may be treated as analogous to the AS model's perceived probability of detection. Rather than failing to *detect* the full tax liability, the tax authority in this case fails to *enforce* the full tax liability. Below we refer to this as the probability of enforcement, π.

Before setting out the debt choice model it is useful to summarise the key relevant AS model results (sub-section 2.1) before adapting those to the late payment context (2.2).

2.1 *Allingham-Sandmo Model Results*

The AS model, as summarised by Sandmo (2005), takes gross income, W, as given, and considers the decision to evade some income, E, from tax, for a given tax rate, t, applied to declared income and a 'penalty tax rate', $F > t$, applied to evaded income if detected. With declared income equal to $W - E$, the taxpayer's objective is to maximise expected utility, V, from income in two possible states – where evasion is undetected (hence net income, $Y = (1 - t)W + tE$) and where evasion is detected (hence net income, $Z = (1 - t)W - (F - t)E$.)

Letting U(X) be utility in state X = Y, Z, and given a perceived probability that the evasion is detected, p, expected utility is given by:

$$V = (1 - p)\mathrm{U}(Y) + p\mathrm{U}(Z) \tag{1}$$

AS show that for a risk-averse taxpayer, an interior solution (in which evasion is positive) is readily obtained from the first order conditions (FOCs). As is well-known following Yitzhaki (1974), the critical condition for this interior solution depends on whether the penalty tax rate is levied on evaded income, E, or evaded tax, tE. Since the latter case is of interest in our enforcement application, we focus on this below. Hence with the penalty for evasion now given by FtE, where $F > 1$, Sandmo (2005, p.647) shows that this yields the first order condition:[2]

$$\frac{U'(Z)}{U'(Y)} = \frac{1-p}{p(F-1)} \tag{2}$$

[2] See also Gahramanov (2009). For the case where the penalty rate is applied to evaded income (F - 1) in (2) is replaced by ($F - t$).

The right–hand side of (2) can be interpreted as the relative price of income in the states of detection ($p(F-1)$) and non-detection ($1-p$), which depends negatively on F and p; see Sandmo (2005, pp.646-7). From (2), some tax evasion is optimal if:

$$pF < 1 \tag{3}$$

That is, the 'expected penalty rate', pF, must be less than 1. It is this result (and its equivalent when the penalty rate is applied to evaded income, rather than the tax owing) that has generated extensive debate over the 'puzzle' that, for most plausible values of p and F, it should be optimal for the vast majority of taxpayers to evade.

For example if the perceived probability of detection is 0.10 (often argued from empirical studies to be an upper bound order of magnitude), then the penalty tax rate, F, would have to exceed 10 before it becomes optimal *not* to evaded tax. That is, if the model is correct, there should be very few non-evaders; a result that seems at variance with general observation. However, as we show below, for the case of New Zealand's late payment penalties, with F around 1.25 there appears to be no such 'puzzle'.

2.2 *Modelling Late Payment Penalties*

Penalties for late payment of tax vary across different countries' tax authorities and often across taxes within the same country. We choose the New Zealand late payment penalty regime in part because it captures elements common to many similar, though often less complex, penalty regimes.[3] In particular we focus on the penalty system used for GST (and income tax, which take the same form). The key components are:

(i) An *initial* penalty in two stages: 1% late payment penalty charged on the day after the due date; and a further 4% penalty charged on amounts of tax and penalties unpaid seven days after the due date. Since indebted taxpayers are almost always in debt for longer than 7 days, below we simplify this penalty as a single 5% initial penalty incurred for paying after the due date.

(ii) An *incremental* penalty charged at 1% per month (approximately 12.7% per year) for outstanding debt including penalties.

(iii) Use-of-money interest (UOMI). Like many countries, New Zealand levies this on all debt inclusive of penalties. The rate varies according to market conditions: currently 8.3% per year since May 2012, reduced from a high of 14.2% in March 2007.[4]

Even with substantial resources committed to enforcing compliance, collection of outstanding tax and penalties rarely reaches 100%. Declaring bankruptcy, for example, generally enables a taxpayer to avoid a past tax liability and can force the revenue authority to consider accepting less than the face value (or its own initial estimate) of an outstanding tax debt. As a result, when an individual owing tax and/or penalties is unwilling or unable to pay immediately, revenue authorities

[3] Like many countries, New Zealand also has a late *filing* penalty, which is fixed as a dollar amount, from \$50 to \$500, differing by the type of tax (e.g. income tax, GST, etc.) and by some taxpayer characteristics (e.g. net income, accounting basis). We ignore this late filing penalty below.

[4] These are 'debit rates', linked to short-term market *borrowing* rates, charged on outstanding debt to IRD. A much lower 'credit rate' is paid on overpayments to IRD, linked to market-based short-term *deposit* rates.

will often consider some penalty reduction or 'discount' to maximise the expected value of the collected tax liability.

A common form of late payment penalty 'discount' used by tax authorities, including in NZ, is to offer the taxpayer an instalment arrangement in which outstanding debt is repaid in discrete instalments over an agreed period. In some cases the reduction may simply be the suspension, or reduced level, of interest payable, and the associated time-cost benefits to the debtor where his/her discount rate exceeds the interest rate payable on debt. In other regimes, as in NZ, selected penalties may be suspended.

In considering the role of reduced penalties in an adapted AS model, we specify an instalment arrangement in which the *incremental* penalty is cancelled on debt which remains outstanding during the arrangement, but where the initial 5% penalty and interest remain payable.

Both the standard penalties and the instalment-related reduction are quite substantial. Letting ϕ represent the initial penalty and f represent the incremental penalty, the *effective* fine rate for the first dollar of debt (the 'extensive margin') is given by $(1 + \phi + f)(1 + r) = 1.276$ or 27.6%, while the 'intensive' *marginal* penalty rate applicable is $(1 + f)(1 + r) = 1.226$ or 22.6%. With $f = 0$ when an instalment arrangement is in place, this reduces the effective 'extensive' and 'intensive' penalties to 13.8% and 8.8% respectively.[5] The percentage effective penalty rates are therefore relatively large but are more than halved in an instalment arrangement.

In the next sub-section we consider an AS type model which allows for less-than-full enforcement of those penalties and the associated outstanding tax liability. To provide orders of magnitude, the model is then illustrated using the structure and values from the NZ GST regime described above.

2.3 *Adapting the AS Model to Late Payments*

This sub-section sets out a simplified two-period model of an individual taxpayer's tax payment choices. As in the AS evasion case, we observe some taxpayers paying on time without penalty while others delay and incur penalties on the outstanding debt. Tax 'avoiders' who are in debt must also decide how much of their total tax liability to delay/avoid. Taxpayers who delay face both an enforcement, and a non-enforcement, state with an associated probability.

In practice, non-enforcement may take several forms such as a full or partial debt write-off (where the tax authority is persuaded that the debt will never be paid, e.g. when there is a prospect of bankruptcy), or a negotiated/legal settlement involving less than full payment, or a further delay in payment.

To proceed, we initially ignore the instalment option, considering only the full penalty case, and assume the taxpayer does not incur any (non)compliance costs in avoiding enforcement of debt payment; i.e. $c_j = 0$ below). Both are introduced subsequently. We define the following terms:

[5] The effective marginal penalty rate in instalments exceeds the interest rate since interest continues to apply to the fixed penalty; i.e. $(1 + r + r\phi)$.

ϕ = initial late payment penalty	f = incremental late payment penalty
r = interest rate applied to overdue tax payments[6]	ρ_j = taxpayer j's 'borrowing rate' (= opportunity cost of each \$ paid in tax at t)[7]
α_j = fraction of j's period t tax liability paid at t under instalment arrangement	π_j = taxpayer j's perceived probability of enforcement of full tax liability ($0 \le \pi_j \le 1$)
c_j = non-compliance cost (per \$ of tax debt) incurred by the taxpayer to avoid enforcement	

Consider an individual taxpayer, j, owing tax to the revenue authority and who must choose whether to pay each dollar of liability in period t, or delay to t+1. The 'price' of paying \$1 at t, rather than t+1, is $(1 + \rho_j)$ – the cost to the taxpayer of borrowing \$1 or the income foregone from (not) saving \$1. The price of delaying payment is the tax plus penalties paid at t+1; namely $(1 + \phi + f)(1 + r)$.[8] The *relative* price of delayed payment is therefore $(1 + \phi + f)(1 + r)/(1 + \rho_j)$.

For convenience below we define the 'total effective penalty rate', or 'fine', as:

$$F \equiv (1 + \phi + f)(1 + r) > 1 \qquad (4)^{9}$$

This fine is paid at t+1 with probability, π_j, when enforced, and with probability $(1 - \pi_j)$ of non-enforcement. Hence the expected fine at t+1 is $\pi_j F$, which is valued by the taxpayer in period t as $\pi_j F/(1 + \rho_j)$.

Following the AS approach above, and taking gross income, W, as given, the individual may decide to delay some tax payment, D, at a cost of FD if payment is enforced. The taxpayer's objective is again to maximise expected utility, V, from income in two possible states – where debt payment is unenforced, U(Y), and where debt payment is enforced, U(Z). Hence:

$$V = (1 - \pi_j)\mathrm{U}(Y) + \pi_j\,\mathrm{U}(Z) \qquad (5)$$

where:

$$Y = W + D \qquad (6)$$

and $$Z = W + D - \{F/(1 + \rho_j)\}D = W - \{(F/(1 + \rho_j)) - 1\}D \qquad (7)$$

It is readily shown that the equivalent to equation (2) for this case becomes:

$$\frac{U'(Z)}{U'(Y)} = \frac{(1-\pi_j)(1+\rho_j)}{\pi_j\{F-(1+\rho_j)\}} = \frac{(1-\pi_j)}{\pi_j\{(F/(1+\rho_j))-1\}} \qquad (8)$$

As in the standard AS case, the right-hand side of (8) can be thought of as the relative price of income in the two states – non-enforcement $(1 - \pi_j)(1 + \rho_j)$, and enforcement $(\pi_j\{F - (1 + \rho_j)\})$.

Analogous to (3), it can be shown from (8) that a necessary and sufficient condition for debt to be optimal is:

[6] We abstract from inflation hence r may be thought of as a real (= nominal) interest rate.
[7] We abstract from the lending-borrowing margin, assuming that taxpayers can borrow from, or lend to the financial system at the same interest rate. This may differ, however, from the interest rate charged by IRD on outstanding debt.
[8] In fact, incremental penalties, f, are levied on *all* outstanding debt, including the unpaid initial penalty, ϕ. Hence strictly this should be specified as $(1 + f)(1 + \phi)$ rather than as $(1 + f + \phi)$. However the missing interaction term, $f\phi$, is small (0.006 at current fine rates) and is ignored below.
[9] Note that under some conventions the fine rate would be defined such that it is the proportional addition to the tax liability e.g. 0.3 (30%) rather than the total liability inclusive of fine, 1.3. Here we follow Allingham and Sandmo (1972) and Sandmo (2005) in defining the 'fine' inclusive of the tax; hence 1.3, rather than 0.3.

$$\pi_j F < (1 + \rho_j) \tag{9}$$

That is, the expected penalty is less than the one plus the 'borrowing rate of interest'. For $\pi_j F > (1 + \rho_j)$, the taxpayer is better off paying the period t tax liability in period t; that is, immediately without penalty, but foregoing the potential rate of return $(1 + \rho_j)$ on unpaid tax.

The condition in (9) allows us to comment on the AS 'puzzle' above, in this context. The AS puzzle arises if it is accepted that there are likely to be many fewer evaders in reality than the model, allied with plausible parameters, would suggest. In our context the equivalent question is whether, given known values of F and plausible values for π_j and/or ρ_j, the model predicts plausible numbers of non-debtors.

New Zealand Inland Revenue (IR) data on the numbers of GST debtors show that in July 2015 around 13% of all GST payers were currently in debt on their GST payments. However, almost half (46.5%) of all registered GST payers had been in debt at some point either currently or in the past. This suggests that many (perhaps half of all) taxpayers would satisfy condition (9) but also half would not.

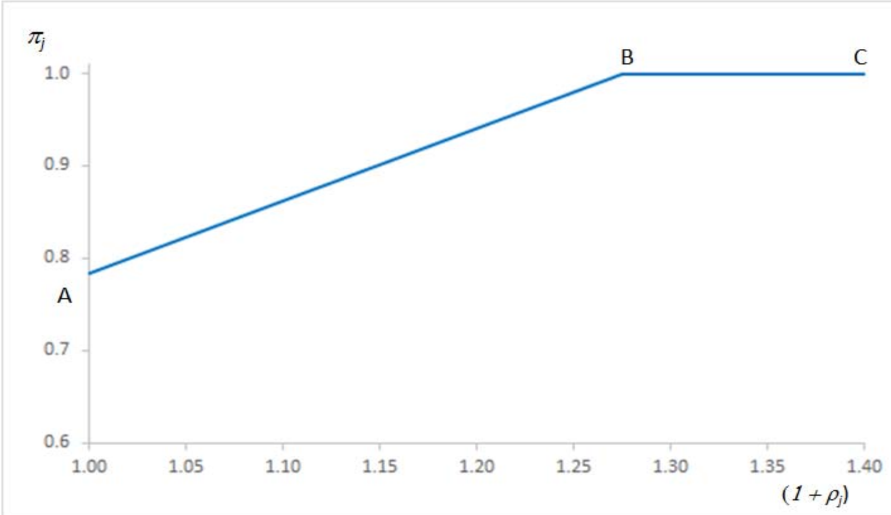
With $F = 1.28$ and a plausible probability of enforcement around 0.85 (85% of outstanding debt is expected to be collected), this implies a critical threshold condition: $1.09 < (1 + \rho_j)$. That is, those for whom the cost of borrowing is less than around 9% would be expected to pay their tax liability on time, while those for whom borrowing cost exceeds 9% prefer to delay payment – i.e. borrow from IR. This threshold ρ_j rises to 15% if $\pi_j = 0.9$.

While most taxpayers may not have ready knowledge of their true probability of enforcement, π, it is likely to be much higher than the equivalent probability of detection in the AS model, since it essentially depends on the tax authority's enforcement practices, rather than ability to detect evasion; see section 3. Unlike evasion, with late payment the amount outstanding is known to both parties, and the IR are generally known to pursue outstanding tax vigorously including using legal powers to auto-seize owed tax directly via banks. As a result, an *average* π_j around 0.8-0.9 seems quite plausible, and the proportion of evaders/debtors observed in this case would seem not to be out of line with plausible empirical predictions from the model.

Using equation (9), the conditions under which debt is preferable to immediate payment are illustrated in Figure 1, using the penalty value above of $F = (1 + \phi + f)(1 + r) = 1.276$. With π_j on the vertical axis and $(1 + \rho_j)$ on the horizontal axis, the line AB, with slope $1/F$, shows points at which the taxpayer is indifferent between the two options: $\pi_j F = (1 + \rho_j)$. At $\pi_j = 1.0$, the line segment BC becomes horizontal.

It can be seen that, at $(1 + \rho_j) = 1$, $\pi_j = 0.784$, and π_j reaches its maximum value of 1 at $(1 + \rho_j)$. $= 1.28$. To the south-east (SE) of the line ABC the taxpayer prefers to delay payment and incur debt, while immediate payment is preferred north-west of the line. For $(1 + \rho_j) > 1.28$ it is always preferable to incur debt – since the taxpayer's cost of borrowing is greater than the effective penalty rate (the effective cost of borrowing from the tax authority).

Figure 1 Optimal Tax Debt Choice



Introducing Non-compliance Costs

It seems reasonable to suppose that avoiding enforced payment of a tax debt is not costless for the taxpayer. Such costs could include the costs of advice from a tax agent, payment of legal or negotiation costs associated with a tax/debt dispute, and setting up structures to prevent seizure of outstanding debt. The taxpayer's willingness to incur such costs might be expected to be related to (a) the amount of debt in question, and (b), the extent to which the probability of enforcement, π_j, could be reduced by incurring such costs. An example of the latter would be where paying for expert advice increases the taxpayers likelihood of negotiating a write-off of the debt, hence reducing π_j. In this section we consider only case (a). The appendix describes case (b) where it is shown that, qualitatively, results are unchanged – essentially the linear relationships in Figures 1 and 2 become non-linear to varying degrees.

Defining those (period t) costs in relation to the size of the tax debt, D, such that $C = \text{c}D$, where $C = 0$ if $D = 0$, and we can rewrite equations (6) and (7) as:

$$Y = W + (1 - \text{c})D \tag{6'}$$

and $$Z = W - \{(\text{F}/(1 + \rho_j)) - (1 - \text{c})\}D \tag{7'}$$

which yields:

$$\frac{U'(Z)}{U'(Y)} = \frac{(1-\pi_j)(1+\rho_j)}{\pi_j\{F'-(1+\rho_j)\}} \tag{8'}$$

where $F' = F/(1 - \text{c})$ is the effective fine rate, adjusted for non-compliance costs. Thus with positive non-compliance costs the condition in (9) simply becomes:

$$\pi_j F' < (1 + \rho_j) \tag{9'}$$

or $$\pi_j < (1 + \rho_j)/F' \tag{10}$$

where $\pi_j F'$ is taxpayer j's 'expected effective fine rate' net of non-compliance costs. Comparing (10) and (9) it is clear that with positive non-compliance costs, other things equal, the critical threshold probability of enforcement, π_j, at which debt becomes optimal is lower, since $F' > F$. That is, the line segment AB in Figure 1 moves down; such that fewer taxpayers in the (π_j, $(1 + \rho_j)$) space in Figure 1 would now consider debt preferable to immediate payment of their tax liability.

Where the taxpayers of interest are those already in debt (as is typically the case when allocation of compliance enforcement resources is being considered), we would expect, other things equal, that such taxpayers would be located below the line ABC in Figure 1. As Allingham and Sandmo (1972) noted in their context, this 'other things equal' condition is important. Individual taxpayers' preferences over whether to evade or not (or, in this case, delay payment) might be expected to be affected by a variety of factors not included in this simple model.

In the absence of conditioning for those other factors, taxpayers might therefore be observed, in terms of Figure 1, to be above the line segment AB, but nevertheless prefer debt to compliance conditional on those other factors. In empirical exercises, therefore, in order to observe the hypothesised relationships between indebtedness, π_j, F and ρ_j, it would be important to control for other influences on tax debt choices. One such factor is likely to be the taxpayer's knowledge of the penalty regime, considered further below.

Introducing a Reduced Penalty Option for Debtors[10]

Now consider the case where only taxpayers already in debt are offered reduced penalties via an instalment option, and which is not anticipated by the taxpayer in making their initial decision to delay payment. This adds two new dimensions to the debt choice problem. Firstly, the applicable penalty is lower but the instalment penalty regime is assumed to be fully enforced. Secondly, instalments involve a mixture of immediate and delayed payment and hence might be expected to reflect some of the properties of both of the previous payment options.

The instalment payment option is simplified here as an agreement by the indebted taxpayer to pay a fraction of the tax owed, α_j, at t, with $(1 - \alpha_j)$ paid at t+1. In this case, the portion delayed to t+1 is liable to a reduced penalty rate of:

$$F_I = (1 + \phi)(1 + r) \tag{11}$$

The taxpayer was previously predicted to be indifferent between incurring debt and paying immediately if, from (10):

$$\pi_j = (1 + \rho_j)/F' \tag{10'}$$

Now an already indebted taxpayer faces a different choice. The indebted taxpayer's expected income under the two options (remaining in debt or agreeing to instalments) can now be expressed as:

$$E[Z_D] = W - \{\pi_j(\mathrm{F}'/(1 + \rho_j)) - (1 - \mathrm{c})\}D \tag{12}$$

and
$$\begin{aligned} E[Z_I] &= W + (1 - \alpha)D - (1 - \alpha)\{F_I/(1 + \rho_j)\}D \\ &= W - (1 - \alpha)\{(F_I/(1 + \rho_j)) - 1\}D \end{aligned} \tag{13}$$

[10] Note that, consistent with IRD practice, the instalment option is modelled here as available only to debtors; i.e. after the decision to 'pay or delay'. If this option was available to (or anticipated by) taxpayers at the point of choosing whether to become indebted or not, it would affect the perceived penalty calculation in a somewhat different manner in the taxpayer's calculus.

where $E[Z_D]$ and $E[Z_I]$ are respectively expected income in the debt and instalment cases respectively.

In (12), expected income is simply gross income, W, plus the value of the debt net of non-compliance costs, $(1 - c)D$, less the value of debt (repaid) inclusive of fines if enforced, $\pi_j(F'/(1 + \rho_j))D$. As previously, enforcement occurs in period t+1, hence is discounted at $(1 + \rho_j)$. In (13), the instalment regime boosts gross income immediately by $(1 - \alpha)D$ – the component of debt not paid immediately by agreement – but income is reduced by $(1 - \alpha)(F_I/(1 + \rho_j))D$ when the second instalment is paid at t+1.

Setting (12) equal to (13) it can be shown that the taxpayer is indifferent between these two options where:

$$\pi_j = (1-\alpha)\frac{F_I}{F} + \frac{(\alpha - c)}{F}(1+\rho_j) \tag{14}$$

Hence taxpayers for whom π_j is greater than the right-hand-side of (14) will prefer to agree to an instalment arrangement with its lower penalties, whereas taxpayers with π_j less than the right-hand-side of (14) will prefer to remain in debt, risking subsequent enforcement.

Comparing (10') and (14) it can be seen that the relationship in (14) between π_j and $(1 + \rho_j)$ is no longer proportional – equation (14) includes a fixed term $(1 - \alpha)F_I/F$ – and now has a lower slope, $(\alpha - c)/F$, compared with $(1 - c)/F$ in (10'). Further, the relationship between π_j and $(1 + \rho_j)$ is positive iff $\alpha > c$. This reflects the fact that higher non-compliance costs involve an opportunity cost in the form of a foregone return at rate, ρ_j, and hence require a lower probability of enforcement at higher ρ_j for the taxpayer to remain indifferent between the two options: continued debt or an instalment arrangement.

As can be seen from (14), if $\alpha = 1$ the instalment option becomes identical to immediate payment; hence $\pi_j = \frac{(1-c)}{F}(1+\rho_j)$ and is identical to (10). If all instalments can be delayed to t+1 ($\alpha = 0$), then $\pi_j = \frac{F_I}{F} - \frac{c}{F}(1+\rho_j)$, and the relationship between π_j and $(1 + \rho_j)$ is unambiguously negative for $c > 0$. In this case the critical condition for the taxpayer to be indifferent is determined by the extent of the penalty discount, F_I/F, together with any non-compliance costs when $c > 0$.

The values of π_j and $(1 + \rho_j)$ at which the taxpayer is indifferent between all three options (immediate payment; remaining in debt; and entering an instalment agreement) can be shown from (10') and (14) to be:

$$\pi_j^* = (1-c)\frac{F_I}{F} \tag{15}$$

and

$$(1+\rho_j)^* = F_I \tag{16}$$

These relationships are depicted in Figure 2, using the effective penalty rates given earlier of F = 1.28 and F_I = 1.14, together with an assumed non-compliance cost parameter, c = 0.1.[11] Using those values yields π_j^*= 0.803 and $(1+\rho_j)^*$ = 1.14.

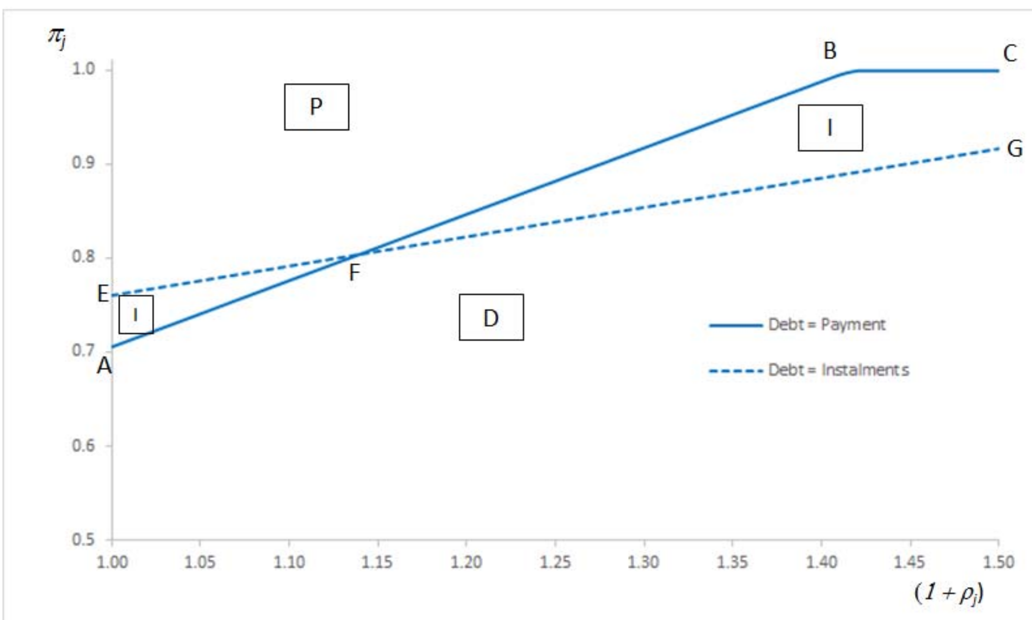
Testing Predicted Responses to Penalties

The above model can help to predict likely responses by taxpayers to the penalty enforcement efforts. As noted above, where taxpayers are in debt, they may be regarded as having chosen a position below the line ABC in Figure 1; that is, having made the prior choice to defer payment of their tax liability. Efforts to encourage compliance such as by initiating greater contact with taxpayers (with or without an offer of reduced penalties via instalments) may be expected to raise their perceived probability of enforcement, π_j, and/or raise the costs, c, associated with further non-compliance. Such interventions aim to shift the taxpayer vertically in Figure 1.

Without an offer of reduced penalties via an instalment arrangement, immediate payment becomes the optimal choice for those shifted above the line AB. Clearly, for taxpayers with relatively high borrowing costs, this is less likely, than for those with lower borrowing costs, with the former being better off in debt at almost all π_j values; see Fig. 1.

When an instalment option is offered to debtors, as in Figure 2, an increase in π_j resulting from an intervention will shift only a sub-set of debtors into the 'instalment preferred' segment of Figure 2 shown by the area, FBCG, marked "I".

Figure 2 Remaining in Debt versus Agreeing to Instalments



The area AEF, to the left of $(1+\rho_j)^*$ = 1.138, is also marked "I" and indicates taxpayers for whom immediate payment is optimal given a choice only between debt with full penalties or on-time payment. However, when an instalment option becomes available to debtors it would be in

[11] Note that the position of the line ABC is shifted downwards in Fig. 2 compared to Fig. 1 due to the positive non-compliance costs, c = 0.1 in Fig. 2, such that, *ceteris paribus*, fewer debtors are expected.

those taxpayers interests to delay payment if they anticipate that an instalment penalty option would then be available. The area AEF is also relevant to those initially in debt when an intervention raises π_j, since a vertical shift in π_j for taxpayers for whom $(1+\rho_j)^* < 1.138$, can either shift their preference into the instalment option or, if sufficiently large, into the payment option (or neither for those who remain below AF).

More generally, Figure 2 suggests that, for a given increase in π_j induced by the enforcement intervention and offer of instalments, those with higher borrowing costs (towards the right in Fig. 2) are more likely to shift to the instalment regime than shift to immediately payment, and *vice versa*.

Penalty Misperceptions

The analysis so far has assumed that the effective penalty rate, F (= $1 + \phi + f)(1 + r)$), is known to taxpayers. But the New Zealand effective penalty system is relatively complex. Even in the simplified version analysed here, there are three elements (ϕ, f and r) combining multiplicatively and where r fluctuates over time with market conditions, while the other elements are generally fixed. With some taxpayers likely to have limited knowledge of the penalties they face, it is useful to consider how knowledge of the penalty regime might affect taxpayers' willingness to resolve their tax indebtedness. In addition, interventions by the tax authority aimed at raising π_j, might also target improvements in taxpayers' knowledge of penalties if this raises compliance.

Clearly, in the absence of full information, taxpayers may under- or over-estimate penalty rates. However, given the large size of the effective penalty and the complexity of the regime, our null hypothesis is that taxpayers who lack full information on the penalty rates, tend to *under*-estimate the effective penalty rate. This rate, of around 27%, is high (perhaps surprisingly high) relative to private market borrowing options.[12]

Figure 3 illustrates the impact of underestimating the penalty rate using the case of $\phi + f = 0.05$ instead of 0.177; equivalent to, for example, being aware of the fixed penalty of 5% but unaware of the 12.7% incremental penalty. This shifts the line ABC in the case of full knowledge, north-westwards to HJC in Figure 3.

Unsurprisingly the combinations of π_j and ρ_j for which immediate payment, rather than delay, is perceived as optimal are reduced – by the area AHJB. Hence, compared to a taxpayer who is fully informed, those who initially underestimate penalties but are subsequently informed of the penalty regime and offered the instalment penalty/payment option as part of a compliance enforcement effort are now much more likely to view the instalment option as optimal – as shown by the two areas FBCG (fully informed taxpayer) and HJCGE (underestimating taxpayer).

Of course, as in the fully informed case, since such an enforcement intervention is also expected to raises π_j, this may encourage immediate payment to the extent that the taxpayer shifts above the line HJ. *Ceteris paribus*, this is less likely compared to a fully informed taxpayer for whom there is a

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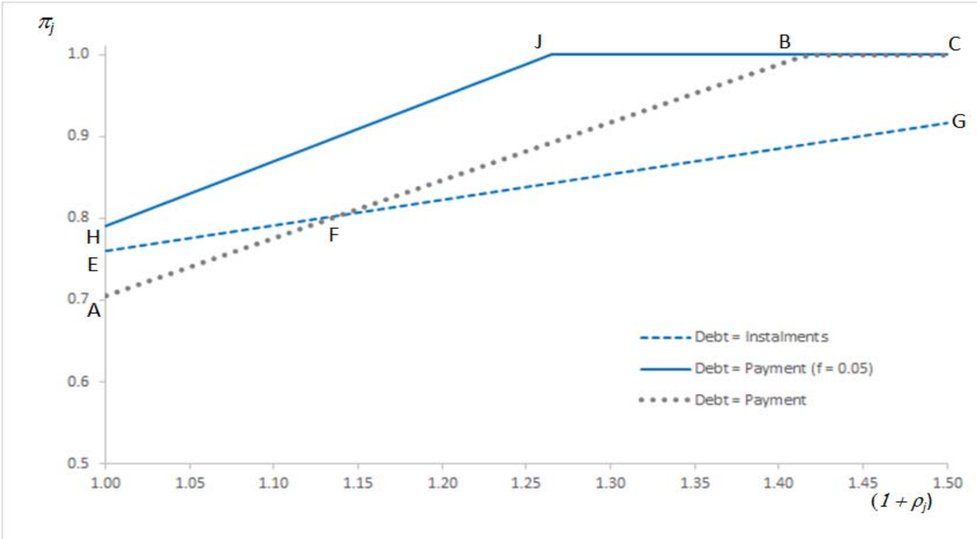
[12] For example typical rates of interest on New Zealand credit cards were around 18-21%, and bank business lending/ overdraft rates around 8-11%, at this time. See http://www.interest.co.nz/borrowing/business-base-rates, for example.

larger area (above AB) where immediate payment becomes optimal when the perceived probability of enforcement increases.

Figure 3 Effects of Penalty Misperceptions



2.4 *Testing the Model*

The preceding analysis leads us to suggest a number of specific hypothesis that may in principle be tested where suitable data on indebted taxpayers are available.

H1: Contact by the tax authority seeking debt repayment, *ceteris paribus*, increases taxpayers' perceived probability of enforcement of their tax debt, π_j, thereby increasing the likelihood of increased compliance; that is, greater willingness either to pay immediately or to enter an instalment arrangement (even if the latter involves no associated penalty reduction).

H2: For taxpayers *with relative high borrowing costs* ρ_j, action by the tax authority which increases their perceived probability of enforcement is more likely to encourage an instalment choice, relative to immediate payment, and *vice versa* for low borrowing cost taxpayers.

H3: Among those with relative high borrowing costs ρ_j, reducing penalties (via the instalment offer) increases the probability that taxpayers will comply via an instalment agreement. But for already indebted taxpayers with low ρ_j., reduced penalties offered via instalments will not increase the probability of choosing an instalment option unless π_j, is simultaneously increased – see Figure 2 for taxpayers with $\rho_j < \rho_j^*$, who initially lie below AF.

H4: For taxpayers underestimating effective penalty rates, providing more accurate information on existing penalties increases the probability that an instalment option with lower penalties will be chosen when offered.

For tax authorities it is often an operational requirement (or preference) that compliance enforcement first encourages debtors to make full and immediate payment of their debts. Thus, only when some 'unaffordability condition' has been established is an instalment option offered.[13] This inevitably applies equally to taxpayers who are ill-informed about penalties. As a result, taxpayers who initially under-estimate penalties and are in the area AHJB in Fig. 3 may agree to pay immediately when informed of penalties since it is in their interests to do so at that stage. They are therefore never offered a reduced penalty instalment option, though it would clearly be preferable to them if offered. This leads to:

H5: Taxpayers given specific (accurate) penalty information are more likely to prefer *immediate payment* if an instalment option is not also offered and cannot be anticipated by the taxpayer.

Finally, note that only where the sample of taxpayers consists of both those observed to be in debt and not in debt can another prediction of the earlier model be tested. Namely, that some taxpayers who would be expected to pay immediately given full knowledge of the penalty regime, may prefer to delay payment if/when they anticipate that an instalment option *for debtors* is available. Those taxpayers are represented by the triangle AFE in Figure 2 and would be expected to shift from a preference for compliance via immediate payment to a preference for non-compliance via instalments.

As with the traditional AS model, a challenge in testing any of those hypotheses empirically is likely to be how to capture or proxy the key subjective variables: the taxpayer's perceived probability of enforcement, potential non-compliance costs associated with avoiding enforcement, and taxpayer-specific borrowing costs. Nevertheless, it is possible that a number of observable taxpayer characteristics could be identified that can be expected to be correlated with the model's more difficult-to-observable characteristics: π_j, c and ρ_j. The next section considers the first of those.

3. Estimating the Probability of Enforcement

The probability of enforcement, π_j, is a crucial element of the adapted AS model in the previous section, and it was argued that the oft-quoted AS 'puzzle' (that estimates of actual evader numbers are much lower than predicted by the model) may not apply in this enforcement context. That argument is based on the presumption that a taxpayer's subjective probability of *enforcement* (once detected) is likely to be much higher than the value typically assumed for a potential evader's probability of detection.

By their nature, these subjective probabilities will vary across taxpayers and are difficult to identify. In the case of detection probabilities, however, the observed fraction of taxpayers who are audited or otherwise subjected to investigation is often taken as an indicator of relevant orders of magnitude. Since these usually vary from less than 1% to around 5% of taxpayers, this is often regarded as a suitable approximation for an 'average' taxpayer's probability of detection.

In the case of the enforcement probability an equivalent proxy might be the fraction of assessed tax liability that is collected. New Zealand Inland Revenue does not publish this by tax

[13] For a UK and two US (North Carolina and Minnesota) examples, see https://www.gov.uk/if-you-dont-pay-your-tax-bill/overview , http://www.dor.state.nc.us/collect/installment.html, and http://www.revenue.state.mn.us/collections/Pages/Payment_Agreements.aspx .

type but does publish various measures across all revenue sources ('receivables'); see Inland Revenue (2016). Table 1 below gives four types of measure of 'recoverable tax'. These are:

(1) 'Impairment receivables' – revenue (at nominal, or 'face', value) owed to IR in a given tax year but which IR does not expect to receive, under standard accounting conventions (e.g. because of firm bankruptcy, court rulings etc.).[14]
(2) Revenue that is 'past due date' (also recorded at 'face value'). That is, the revenue is, at best, expected to be paid late – hence the taxpayer potentially gains in present value terms.
(3) Impairment losses and debt written off in a given tax year. This differs from (1) in that it relates only to the *flow* in the impaired tax in the current year.
(4) 'Recoverable' tax debt, estimated at net present values (NPV).

Table 1 shows IR (2016) estimates of those four categories for two tax years, 2014/15 and 2015/16, in dollar values and as percentages of 'gross receivables'.

Table 1 Estimates of Recoverable Overdue Tax

		($000s) **2014-15**	($000s) **2015-16**
1.	Receivables		
	Gross receivables	12,145,147	12,607,405
	Impairment receivables	4,192,162	3,745,898
	Carrying value receivables	7,952,985	8,861,507
	Impairment (% gross receivables)	**35%**	**30%**
	Impairment (% past due date)	**81%**	**80%**
2.	Age profile of gross receivables		
	Not yet due	6,992,052	7,927,376
	Past due date	5,153,095	4,680,029
	Past due (% gross receivables)	**42%**	**37%**
3.	Receivables - impairment		
	Balance at 1 July	4,466,435	4,192,162
	Impairment losses recognised (+)	860,829	680,343
	Amounts written off as uncollectable (-)	1,135,102	1,126,607
	Balance at 30 June	4,192,162	3,745,898
	Impairment losses (% gross receivables)	**7%**	**5%**
	Uncollectable (% gross receivables)	**9%**	**9%**
	Impairment losses (% carrying value receivables)	**11%**	**8%**
	Uncollectable (% carrying value receivables)	**14%**	**13%**
4.	Recoverable receivables	NPV ($000s)	NPV ($000s)
	Recoverable receivables not yet due	6,954,717	7,892,385
	as % nominal value*	**99.5%**	**99.6%**
	Recoverable receivables past due date	998,268	969,122
	as % nominal value**	**19%**	**21%**
	as % gross receivables	**8%**	**8%**
	UOMI interest rate used	9.2%	8.3%
	Discount rate used	6%	6%
	Impact on recoverable amount of 2% decrease in discount rate	21,000	30,000

Notes: * Gross receivables not yet due; ** Gross receivables past due date.
Source: Inland Revenue (2016, pp. 120-1).

[14] Receivables (whether impaired or unimpaired) include any added penalties and interest (UOMI).

Measure (1) shows that impaired receivables represent around 30-35% of total receivables (revenues), in part reflecting the large amounts of debt accumulating from past unpaid tax, penalties and interest (UOMI) that is no longer considered collectable. The age profile of gross receivables in Table 1 also shows that the overdue revenue measure (2) is somewhat higher at around 37-42% of total receivables.

Measures (3) and (4) are perhaps most relevant to the 'enforcement probability' issue since (3) is a flow, rather than stock, and (4) is a direct estimate of recoverable revenue measured in NPV terms. Table 3 suggests, for example, that in the two tax years reported impaired losses were around 5-7% of gross receivables, with an additional 9% considered unrecoverable and written-off (hence in accounting terms it is removed from the estimated debt stock).

Similarly, the estimate of recoverable receivables in present value terms (measure 4) was only around 20% of its nominal value when those receivables were already overdue, or around 8% of gross receivables. When combined with expected recovery of receivables that are not yet overdue (at 99%), the overall recoverable NPV is around 65-70% of gross receivables. That is, as with the 'face value' measure (1), around 30-35% is considered unrecoverable.

These data suggest that in general a much higher degree of enforcement might be expected by taxpayers than the traditional view regarding expected detection of evasion. However, it also appears to be the case that taxpayers in general might reasonably expect a relatively high fraction, perhaps up to 30-35%, of their tax liability to be uncollected especially for taxpayers whose liability becomes overdue. Of course, there are nevertheless many reasons why a given taxpayer would not want to get into this 'overdue' position, as demonstrated by the earlier model, and from the GST payer data discussed above – namely, only around 13% of GST payers were in debt to IR at a given point in time (July 2015) and under 50% had been in GST debt at some point in the past.

4. Conclusions

This paper has considered how the traditional Allingham-Sandmo (AS) model of tax evasion may be adapted to incorporate less than perfect enforcement of evaded tax. Specifically it examined the case where evasion (more accurately, 'avoidance') takes the form of *late payment* of tax liabilities for which penalties apply. It was argued that the case of late payment avoidance and the penalties associated with it, provides a natural analogue of the original AS evasion context. Standard AS results follow through here if their 'probability of detection' (which is redundant in the late payment context) can be replaced by a 'probability of enforcement' of the delayed tax. In this case the 'expected penalty' plays a similar role to its role in the original model, but the 'price' of detected evasion is no longer the penal tax rate applied to the discovered income, but the 'price' (an interest rate) associated with borrowing from the private market when late payments are enforced.

The paper then explored how reduced penalty incentives for tax debtors, and misperceptions of the penalty regime, can be expected to affect late payment non-compliance. The penalty reduction here has no equivalent in the original AS model though reporting incentives for evaders, such as a tax amnesty, is an example of a similar mechanism in the evasion context.[15] An important aspect here is that the penalty reduction is only available, and *perceived as available*, after the taxpayer has first made the decision to pay late (or paid late by default), hence incurring a tax debt. Otherwise a fully informed taxpayer's utility maximisation exercise in the first-stage decision would be constrained merely by the reduced penalty. Absent this, it was shown that a reduced penalty *associated with debt repayment instalments* is expected, *ceteris paribus*, to encourage a subset of indebted taxpayers to comply fully – via immediate full payment – while a different sub-set comply partially via instalments. Examining the impact of under-estimation of penalties, it was shown that, as well as increasing indebtedness as would be expected, this can also affect the indebted taxpayer's *relative* preference for instalments versus full repayment of debt.

The adapted AS model generated a number of specific, testable hypotheses that were set out in the previous section. In general, these hypotheses are testable on samples of indebted taxpayers; that is, those who have made the prior decision to pay late (whereas testing the equivalent here of the AS 'decision to evade' requires data on both indebted and non-indebted taxpayers). It was argued that, since taxpayer responses to enforcement are predicted to differ according to their values of π_j, c_j and ρ_j, suitable taxpayer-specific proxies for those could potentially allow the various hypotheses to be tested, providing an avenue for future empirical research on the impact of late payment penalties and compliance enforcement.

Finally, New Zealand data on potentially 'unrecoverable' tax, considered in section 4, suggested that an 'average' value for this proxy for π_j across the tax system as a whole could be as low as 65-70%. This may partly reflect the high penalty and interest rates in New Zealand that can lead to a relatively rapid accumulation of total tax debt once the initial tax assessment becomes overdue.

[15] See, for example, Alm and Beck (1990, 1991), Stella (1991), Malik and Schwab (1991), López-Laborda and Rodrigo (2003), and Farrar and Hausserman (2016).

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APPENDIX Compliance Avoidance Costs and the Enforcement Probability

A taxpayer's willingness to incur costs to facilitate late, or non-payment of tax liabilities might be expected to be related to their perceptions of how far incurring such costs would ensure a lower probability that the tax liability will be enforced by the revenue authority. Earlier in the text that aspect was ignored, instead assuming that such non-compliance costs were simply proportional to the amount of outstanding tax (debt). Here we allow for the possibility that non-compliance costs are (negatively) related to the probability of enforcement.

Consider a situation in which the taxpayer believes that the probability of enforced payment π_j = 1, if no costly non-compliance activity aimed at reducing it is undertaken. Incurring such cost is, however, expected to lead to a lower π_j. For example, where non-compliance costs are perceived to yield a probability that a dollar of debt is collected of 0.8 (expected liability = \$0.80) the taxpayer can incur costs up to \$0.20 (to help achieve this outcome) and still be better off; whereas with a probability of enforcement of 0.6 (expected liability = \$0.60), the taxpayer is better off up to a non-compliance cost per dollar of debt of \$0.40.

Given a fine rate of *F* applied to each dollar of debt, this can formally be expressed as:

$$\pi_j F + c \leq F \tag{A1}$$

or

$$c \leq (1 - \pi_j)F \tag{A2}$$

This suggests a convenient expression for the relationship between c and, π_j as:

$$c = \gamma(1 - \pi_j)F \qquad \text{where } \gamma \leq 1 \tag{A3}$$

which allows equations (6') and (7') above to be rewritten as:

$$Y = W + \{1 - \gamma\big(1 - \pi_j\big)F\}D \tag{A4}$$

and

$$Z = W - [\frac{F}{(1+\rho_j)} - \big\{1 - \gamma\big(1 - \pi_j\big)F\big\}]D \tag{A5}$$

From the FOCs for this amendment to the standard optimisation problem described earlier, (8) now becomes:

$$\frac{U'(Z)}{U'(Y)} = \frac{(1-\pi_j)[1-\gamma(1-\pi_j)F]}{\pi_j\{\frac{F}{(1+\rho_j)} - [1-\gamma(1-\pi_j)F]\}} \tag{A6}$$

Solving for π_j, with $U'(Z) = U'(Y)$, it can be shown that:

$$\pi_j = \frac{F^{-1}-\gamma}{(1+\rho_j)^{-1} - \gamma} = \frac{(F^{-1}-\gamma)(1+\rho_j)}{1 - \gamma(1+\rho_j)} \tag{A7}$$

From (A7) it can be seen that if γ = 0, such that there are no non-compliance costs, (A7) reduces to the earlier expression:

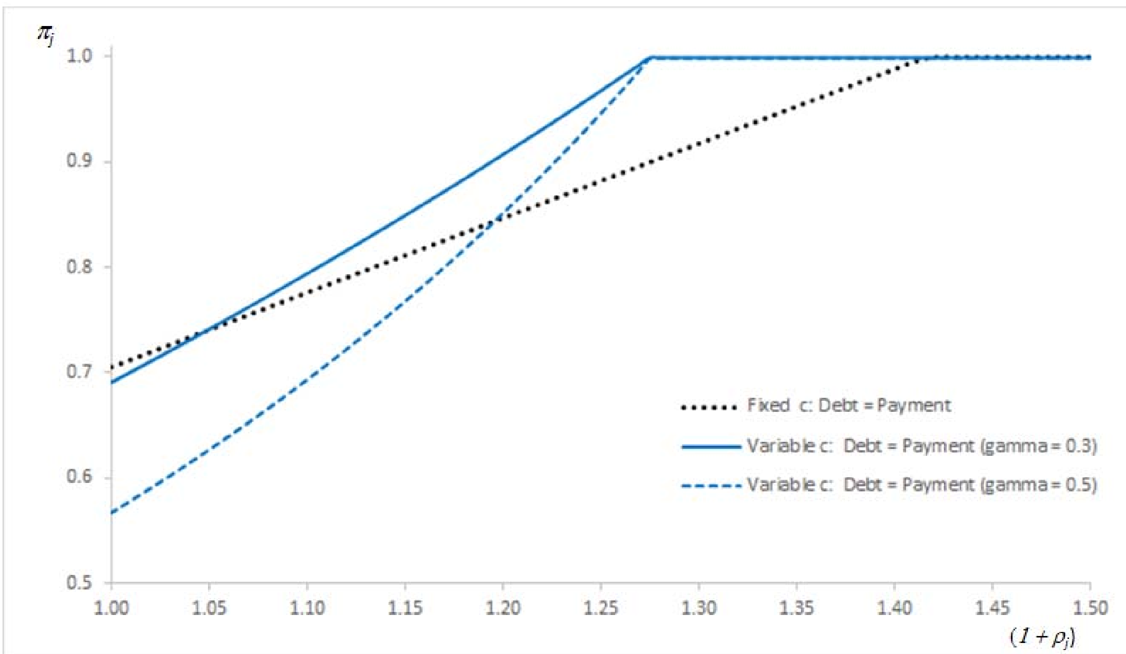
$$\pi_j = \frac{1}{F}(1 + \rho_j) \tag{A8}$$

while if γ = 1 (the taxpayer incurs the maximum costs consistent with expecting to be no worse off after incurring such costs), then:

$$\pi_j = \left(\frac{F-1}{F}\right)\left(\frac{1+\rho_j}{\rho_j}\right) \tag{A9}$$

The fixed, and enforcement probability-related, cases of non-compliance costs are illustrated in Figure A1 for the fixed case of c = 0.10 and the variable cases set at γ = 0.3 and 0.5. For the three cases each line divides the space into combination of π_j and $(1 + \rho_j)$ for which either delayed payment (= debt) or immediate payment is preferred – immediate payment being preferred above/left of the relevant line. The broken black line repeats the illustration in Figure 1, with a slope of $1/F$; see (A8). The two blue lines indicate the two variable non-compliance cost cases (γ = 0.3, 0.5). These can be seen to be similar to the c = 0.1 case but are non-linear, the degree of non-linearity being greater the larger is γ.

Figure A1 Variable versus Fixed Non-Compliance Costs



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