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Optimal Tax Enforcement: Keen and Slemrod Explored*

John Creedy†

Abstract

Keen and Slemrod (2016) provide a framework for examining optimal tax enforcement in an income tax context. This combines the well-known elasticity of taxable income with an enforcement elasticity of taxable income. They derive a number of insightful general results that essentially involve first-order conditions for maximising a 'social welfare function'. The aim of this note is to provide a more elementary derivation of their main results and to produce a number of extension. The model is extended to allow for a direct effect on labour supply of tax enforcement. The single-person model of Keen and Slemrod is then extended to the many-person context. The paper finally introduces some simple functional forms in order to consider closed-form solutions. This illustrates not only how the model can be solved in practice, but helps to reveal some important properties that may not be immediately clear.

JEL Classification:

Keywords: Tax enforcement; elasticity of taxable income; optimal tax.

*I have benefited from discussions with Norman Gemmell, Matt Benge, Richard Braae, Phil Whittington, Hamish Slack and Josh Teng.

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1 Introduction

Keen and Slemrod (2016) provide a framework for examining optimal tax enforcement in an income tax context. This combines the well-known elasticity of taxable income (with respect to the net-of-tax marginal income tax rate) with their 'enforcement elasticity of taxable income' (with respect to an enforcement parameter that influences the cost of income concealment faced by individuals). An advantage of the framework is that it can be closely related to existing optimal tax models in which the elasticity of taxable income plays a prominent role. In those models, the government's optimality condition can be expressed in general terms of an equimarginal principle (relating the marginal cost of raising revenue, including the excess burden, with the marginal 'social value' attached to the resulting public expenditure).[1]

The advantage of their framework and approach is that it enables them to 'cut through' many complexities involved in thinking about optimal tax and enforcement to derive a number of insightful general results that essentially involve first-order conditions for maximising a 'social welfare function'. Such first-order conditions, while elegant, often conceal some important structural relationships in the model and the way in which solutions are obtained for explicit specifications of functional forms. A comparison may be made with the smooth movement of the hands on a clock, where the face conceals a complex set of gears working to turn the hands appropriately.

A major aim of this note is to provide a more elementary derivation of their main results, and in so doing it moves behind the 'clock face' to examine the 'gears', for those who may be interested in applying or extending the basic model. The presentation by Keen and Slemrod (2016) concentrates on discussing the implications of results and their relation to earlier literature, and thereby moves very rapidly over the analytics.

Several extensions are also provided. For each of the first-order conditions, relating

[1] For details and applications to New Zealand, see Creedy (2015). Using a different approach, more closely related to the earlier work on optimal income taxation associated with Mirrlees (1971), Tuomala (1985) produced (for a linear tax) an elegant expression for the first-order condition, involving the elasticity of average income with respect to the tax rate and an inequality measure expressed as the proportional difference between average income and a welfare-weighted mean: see also Creedy (2011, pp. 45, 73-83).

to the optimal tax rate for given enforcement parameter, and optimal enforcement parameter for given tax rate, intuitive explanations are given in terms of equi-marginal conditions relating marginal costs and benefits of changing the relevant variable. The paper also provides an extension of the model by introducing the type of behavioural response to enforcement stressed by Gemmell and Hasseldine (2014), who argue that enforcement activity is likely to reduce labour supply. This is also found to have implications for the optimal compliance gap. Finally, the single-person model of Keen and Slemrod is extended to cover a many-person case. The government's objective is suitably modified to allow for a concern for distributional outcomes, and a basic income is added to provide a progressive tax-transfer scheme. Modified first-order optimality conditions are derived in terms of relevant aggregates and welfare-weighted aggregates.

Section 2 sets out the details of the Keen-Slemrod model, in which a representative individual maximises utility by making a choice about labour supply and the amount of income to conceal from the tax authorities, while the government selects the tax rate and an 'enforcement parameter' to maximise a social welfare function. Both tax administration and evasion are costly, and net revenue is used to finance public expenditure which is valued (at the margin) more highly than private expenditure.

Section 3 introduces the labour supply effect of tax enforcement, which is absent from the Keen-Slemrod model. In their model an increase in enforcement reduces the extent to which income is concealed from the tax authorities, while there is no direct effect on labour supply. Section 4 goes on to consider the optimal compliance gap and the way in which this is affected by labour supply responses to tax enforcement. The extension to the many-person case is explored in Section 5.

Section 6 solves the basic KS model using some simple functional forms in order to consider closed-form solutions, rather than simply the general necessary conditions in more details: more flexible functional forms can be seen to produce rather intractable nonlinear equations. This illustrates not only how the model can be solved in practice, but helps to reveal some important properties that may not be immediately clear. Brief conclusions are in Section 7.

2 The Basic Model

The basic form of the Keen and Slemrod (2016) model – hereafter referred to as KS – contains a single or 'representative' individual who makes choices regarding hours of work and the amount of income to conceal, in order to maximise a quasi-linear utility function expressed in terms of net income (consumption of a private good), the disutility from work and the utility obtained from public expenditure. (They later discuss the extension to many individuals with different wage rates). The individual faces a fixed wage rate, and a proportional income tax rate applied to all declared or 'taxable income', along with a cost of concealing income which depends on the amount concealed and a government 'tax enforcement' parameter.

Taxation in the model essentially involves shifting consumption from the private good to the public expenditure, via income tax revenue (that is, the revenue remaining after administrative costs of enforcement have been incurred). Despite an assumption that the individual's marginal utility from public expenditure exceeds marginal utility from the private good, the high excess burden of taxation means that the individual 'resists' this transfer to some extent.

The 'government' is considered to maximise a social welfare function expressed in terms of the individual's utility, subject to a government budget constraint relating public expenditure to tax revenue less enforcement costs. The framework has the same 'dual decision' nature of the standard optimal tax model, in that the individual optimises subject to a budget constraint which is in turn influenced by 'parameters' which, from the point of view of the government, are the decision variables. The complexity of this kind of model means that explicit or 'closed form' solutions cannot generally be obtained, or at least cannot be obtained without making very strong simplifying assumptions. The challenge – as in all modelling – is to select assumptions which nevertheless retain useful insights into the specific problems raised.

The variables used in the model are set out in Table 1. Some of the KS notation has been changed here.[2]

[2] In particular, the symbol, h, replaces ℓ. This can too easily be confused with e when writing

Table 1: Variables in the Keen-Slemrod Model

Variable	Definition
w	Wage rate
x	Private consumption
h	Hours worked
$\phi(h)$	Disutility arising from working
s	Public non-enforcement spending
α	Tax enforcement parameter
e	Amount of income 'concealed' by individual
c	Expenditure associated with concealment: $c = c(e, \alpha)$
a	Administrative cost of enforcement: $a = a(\alpha)$
z	Taxable income: $z = wh - e$
$v(s)$	Utility from public non-enforcement spending
t	Proportional income tax rate

2.1 The Individual's Problem

The individual is assumed to choose hours worked, h, and the amount of income concealed from the tax authorities, e, in order to maximise the following quasi-linear utility function, U:

$$U = x - \phi(h) + v(s) \tag{1}$$

where x represents private consumption (net income), $\phi(h)$ is the disutility from work, and $v(s)$ is the utility obtained from the tax-financed public expenditure, s.[3] Consider the individual's indifference curves in x-h space given by:

$$dU = dx - \frac{\partial \phi(h)}{\partial h} dh = 0 \tag{2}$$

by hand. Also, KS use ϕ to denote the elasticity of taxable income with respect to the enforcement parameter, α, but here ϕ replaces KS's φ to denote the disutility from work. In writing elasticities, KS use, say, $E(a, b)$ to denote the elasticity of a with respect to b: here this is denoted $\eta_{a,b}$.

[3] The quasi-linear utility function that generates the 'standard' constant elasticity of taxable income of η, and an absence of income effects, is given by: $U = x - \theta z^{1+\frac{1}{\eta}}$. Hence KS extend this to include the benefit of public expenditure and a cost of concealing income which affects the relationship between consumption and earnings. In the standard model, taxable income, z, reflects the 'cost' of obtaining consumption, whereas the KS model adds endogenous labour supply explicitly. Hence KS add a substantial amount in a succinct manner. The use of a quasi-linear utility function with constant marginal utility of one good (here, x) was first adopted in economics by Marshall, in his debate with Edgeworth on exchange and the indeterminacy of prices.

so that the marginal rate of substitution between consumption (net income) and work is:

$$\left.\frac{dx}{dh}\right|_U = \frac{\partial\phi(h)}{\partial h} \tag{3}$$

Consumption is equal to gross income, wh, less taxation (based on taxable income, $wh - e$) less the cost of concealing part of that income, $c(e, \alpha)$. Hence, the individual's budget constraint is given by:

$$x = wh - t(wh - e) - c(e, \alpha) \tag{4}$$

with a slope of:

$$\frac{dx}{dh} = w(1 - t) \tag{5}$$

Equating (3) and (5) gives the first-order condition for maximum utility as:

$$\frac{\partial\phi(h)}{\partial h} = w(1 - t) \tag{6}$$

Diagrammatically, this is of course a tangency position between an indifference curve and budget line, as illustrated in Figure 1, as the point E. The marginal rate of substitution in consumption between h and x along an indifference curve (the ratio of marginal utilities), is equal to the rate at which they can be substituted in the market (maintaining a fixed budget). In the KS model, the marginal utility of x is always equal to 1, so the condition reduces to the simple requirement that the marginal disutility from working is equal to the net (after tax) wage.[4]

This specification implies that labour supply depends only on the disutility of work and the net wage, $w(1 - t)$. It is not affected by the nature of the government's tax enforcement. As stressed by Keen and Slemrod, this property, that the individual does not respond to greater enforcement by working fewer hours, therefore bypasses the kind of response discussed by Gemmell and Hasseldine (2014).[5] This is considered further in Section 3.

[4]The marginal rate of substitution is in general $\left.\frac{dx}{dh}\right|_U = \frac{\partial U}{\partial h} / \frac{\partial U}{\partial x}$. Here $\frac{\partial U}{\partial x} = 1$ and $\frac{\partial U}{\partial h} = \frac{\partial\phi(h)}{\partial h}$.

[5]Such responses would arise in the case of, for example, a Cobb-Douglas type of utility function rather than the directly additive form above. However, this would considerably complicate the KS model.

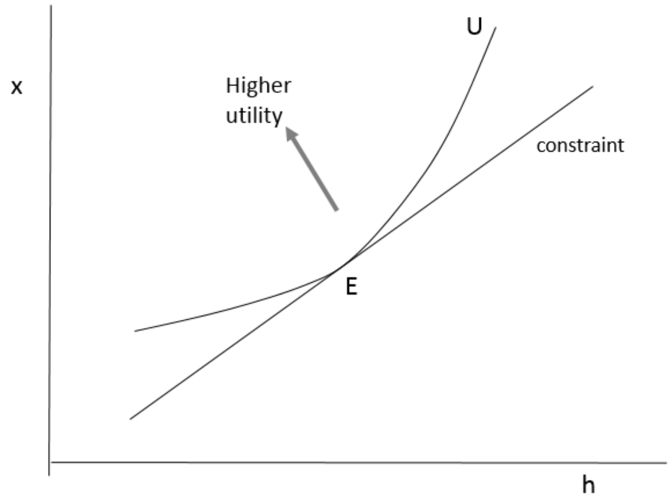

Figure 1: The Individual's Optimum Position

Keen and Slemrod substitute (4) into (1) and differentiate partially with respect to h and e. Setting the first partial, $\partial U/\partial h$, equal to 0 gives (6) and, for the second partial derivative:

$$\frac{\partial U}{\partial e} = t - \frac{\partial c\left(e, \alpha\right)}{\partial e} = 0 \tag{7}$$

giving the first-order condition:

$$t = c_e \tag{8}$$

where by definition, $c_e = \frac{\partial c(e,\alpha)}{\partial e}$. The amount of income concealed from the tax authorities is increased up to the point where the marginal cost of concealement, c_e, is equal to the marginal benefit, or tax saved, t.

2.2 The Government's Problem: Choice of Tax Rate, t

In this 'representative person' economy, suppose the objective is simply to maximise the individual's utility. Here public expenditure of s is effectively a private good (rather than public good), and the government values the expenditure in the same way as the individual. Hence the 'social welfare function', W, is equivalent to U. The government must simultaneously choose the optimal tax rate, t, and the optimal degree of tax enforcement, α. It is convenient to consider two separate choices, where the first problem is to select the optimal t for a given value of α, and the second problem is to optimise α for a given t. The resulting two (nonlinear) equations can then be solved for the optimal combination of t and s.

First, consider maximisation of W by a suitable choice of t and s, for fixed α.[6] Consider 'social' indifference curves in s-t space. Differentiating W totally gives:

$$dW = -(wh - e)\,dt + \frac{\partial v}{\partial s}ds \tag{9}$$

setting this equal to zero, letting $v' = \frac{\partial v}{\partial s}$, and remembering that $z = (wh - e)$

$$\left.\frac{ds}{dt}\right|_W = \frac{z}{v'} \tag{10}$$

The term v' is described by Keen and Slemrod as the 'marginal social value of government spending' (excluding tax administration costs). The government's budget constraint is given by:

$$s = t(wh - e) - a(\alpha) \tag{11}$$

This assumes that there is no other expenditure of a type which does not enter the representative individual's utility. The slope of the budget constraint, letting $z_t = \partial z/\partial t$, is:

$$\frac{ds}{dt} = z + tz_t \tag{12}$$

Clearly, addition a fixed additional component of government expenditure that does not enter the individual's utility function (and thus the government's welfare function) has no effect on first-order conditions.

Equating (12) with (10) gives the first-order condition:

$$\frac{z}{v'} = z + tz_t \tag{13}$$

or:

$$v'(z + tz_t) - z = 0 \tag{14}$$

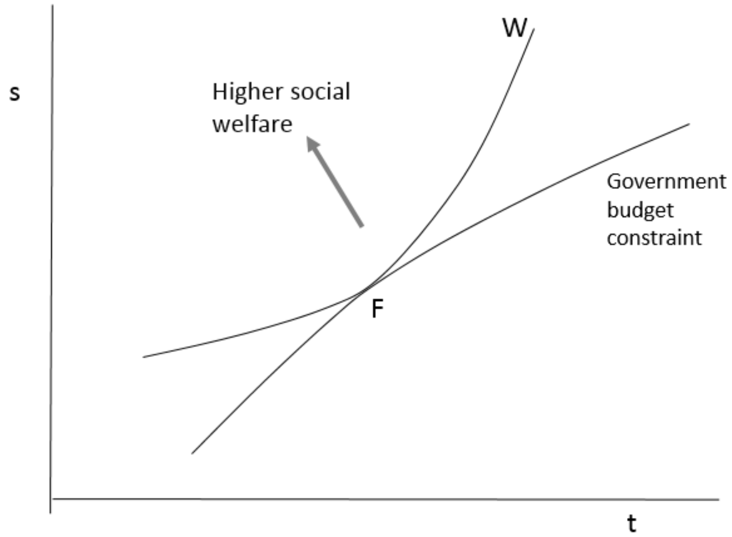

Figure 2: Maximum Social Welfare

This corresponds to Keen and Slemrod, equation (5). The tangency position is illustrated as the point F in Figure 2.

The result in (14) can be rearranged to give:

$$\frac{1}{v'} = 1 + t\frac{z_t}{z} \tag{15}$$

$$= 1 - \left(\frac{t}{1-t}\right)\frac{\partial z}{\partial(1-t)} \tag{16}$$

and using $\eta_{z,1-t} = \left(\frac{t}{1-t}\right)\frac{\partial z}{\partial(1-t)}$ to denote the elasticity of taxable income, this gives:

$$\frac{t}{1-t} = \left(\frac{v'-1}{v'}\right)\frac{1}{\eta_{z,1-t}} \tag{17}$$

This is equation (7) of Keen and Slemrod: it may be described as their first fundamental or major result.

It is possible to rearrange (17) as:

$$t = \frac{v'-1}{v'\left(1+\eta_{z,1-t}\right)-1} \tag{18}$$

[6]Later KS consider a many person economy where individuals face different wage rates, so that the elasticity of taxable income is a suitably weighted aggregate.

However, care is needed as this is not a closed-form solution for the optimal tax rate, as in general v' and $\eta_{z,1-t}$ are endogenous.[7] Nevertheless, insights can often be obtained from first-order conditions such as (17) without specifying functional forms.

2.2.1 Some Intuition

The expression in (17), despite being familiar from other tax contexts, is not readily amenable to an intuitive explanation. It is useful to return to its form in equation (15), since this can be rewritten as:

$$\frac{1}{v'} = 1 + \eta_{z,t} \tag{19}$$

where $\eta_{z,t}$ is the elasticity of taxable income with respect to changes in the tax rate, t, rather than the net-of-tax rate, $1 - t$, and is expected to be negative.

This condition can in fact easily be extended to the slightly more general case where, instead of x appearing additively in the utility function so that its marginal utility is 1, utility is written as $U = m(x) - \phi(h) + v(s)$. Letting $m' = \partial U / \partial x$, it can be shown that (19) becomes:

$$\frac{m'}{v'} = 1 + \eta_{z,t} \tag{20}$$

Consider total income tax revenue, $R = tz$. Differentiating with respect to t gives the well-known result that:

$$\eta_{R,t} = 1 + \eta_{z,t} \tag{21}$$

where $\eta_{R,t}$ is the elasticity of revenue with respect to the tax rate. This expression shows how the revenue change depends on separate 'tax rate' and 'tax base' effects: for the simple proportional tax system the tax rate effect is 1, so that without any behavioural responses revenue would increase in the same proportion as the tax rate. Combining (20) and (21) gives:

$$m' = v' \eta_{R,t} \tag{22}$$

Hence, for a given value of α, the tax rate is increased until the proportional revenue increase, valued in the hands of the government to spend on the public good, s, and

[7] Keen and Slemrod do not give equation (18), but obviously use it to produce their numerical example.

thus giving rise to a marginal utility of v', just matches the marginal utility, m', of private consumption, x. Hence (22) turns out to be a familiar kind of equi-marginal condition. Notice also that income tax revenue is maximised when $dR/dt = 0$, or when $\eta_{z,t} = -1$, and for higher tax rates $\eta_{R,t} < 0$. Hence the tax rate would not be raised to, or beyond, this revenue maximising point: only the range where $\eta_{R,t} > 0$ is relevant.[8]

2.3 The Government's Problem: Choice of Enforcement, α

This subsection turns to the problem of maximising W by a suitable choice of the parameter, α, where the tax rate is held constant. From the welfare function, W, totally differentiating with respect to s and α gives:

$$dW = -c_\alpha d\alpha + v' ds \tag{23}$$

with $c_\alpha = \frac{\partial c(e,\alpha)}{\partial \alpha}$. Hence the corresponding social indifference curves have slope:

$$\left.\frac{ds}{d\alpha}\right|_W = \frac{c_\alpha}{v'} \tag{24}$$

Furthermore, the appropriate slope of the government's budget constraint, $s = tz - a$, is:

$$\frac{ds}{d\alpha} = tz_\alpha - a_\alpha \tag{25}$$

with $z_\alpha = \frac{\partial z}{\partial \alpha}$ and $a_\alpha = \frac{\partial a(\alpha)}{\partial \alpha}$. Equating these slopes gives the tangency position, or first-order condition, as:

$$tz_\alpha - a_\alpha = \frac{c_\alpha}{v'} \tag{26}$$

which becomes:

$$c_\alpha = v' \left(t\frac{\partial z}{\partial \alpha} - a_\alpha \right) \tag{27}$$

and writing $\eta_{z,\alpha} = \frac{\alpha}{z}\frac{\partial z}{\partial \alpha}$ as the 'enforcement elasticity', this gives the result that:

$$\eta_{z,\alpha} = \alpha \left(\frac{\frac{c_\alpha}{v'} + a_\alpha}{tz} \right) \tag{28}$$

[8] Of course, in the absence of behavioural responses to tax rate increases, $\eta_{R,t} = 1$ and the tax rate would be raised to the point where $m' = v'$. In the KS model, in addition, $m' = 1$, so that with $v' > 1$, the optimal rate is pushed to its extreme of 1.

The term, $\frac{c_\alpha}{v'}$, represents the marginal cost of evasion (which Keen and Slemrod refer to as the marginal 'compliance cost'), valued in social valuation terms and thus deflated by v': the marginal private cost is simply c_α. The term a_α is the marginal administration cost. Both these marginal costs are in terms of the extra cost resulting from a marginal change in the parameter, α. Equation (28) corresponds to Keen and Slemrod's equation (9): it may be described as the second fundamental or major result.[9]

Hence the optimal enforcement parameter, α, is such that the elasticity of taxable income with respect to α is proportional to the ratio of sum of the marginal cost of evasion (in social welfare terms) and the enforcement cost, to the total tax revenue, with a constant of proportionality given by α itself.[10] Again, this cannot be rearranged to give a closed-form solution for α, without imposing more structure on the model.

This first-order condition is expressed in terms of the enforcement parameter which influences the individual's cost of concealing income. Keen and Slemrod do not consider the *cost* of imposing or achieving any given value of α and, hence, the implications for the government budget constraint. It would be possible to rewrite the model in terms of expenditure, but this is not considered here.[11]

2.3.1 Some Intuition

It has been seen that the first-order 'enforcement' condition, in (28), has a straightforward interpretation in recognisable terms. But a more intuitive appreciation can be obtained, as in the case of the optimal tax rate first-order condition, by rearranging it into a simple equi-marginal condition. Thus, rewrite (28) as:

$$v' \left(\frac{t \partial z}{\partial \alpha} \right) = c_\alpha + v' a_\alpha \tag{29}$$

[9]KS also point out (p. 6, note 12), that this condition is not affected when a proportional tax is replaced by a tax structure having a tax-free threshold.

[10]KS mention a very simple, but acknowledged to be unrealistic, case where, for example, $c(e, \alpha) = k_1 \alpha$ and $a(\alpha) = k_2 \alpha$, where k_1 and k_2 are parameters. Hence $\alpha c_a = c(\alpha)$ and $\alpha a_\alpha = a(\alpha)$, and the right-hand side of (28) becomes simply the total cost of evasion and enforcement divided by the total tax revenue.

[11]However, (28) can be expressed in terms of the government expenditure on enforcement, q say. Write $\alpha = \alpha(q)$ to express the way in which α depends on expenditure. Then $\eta_{z,q} = \eta_{z,\alpha} \eta_{\alpha,q}$ and (28) can be rewritten as: $\eta_{z,q} = \alpha \eta_{\alpha,q} \left(\frac{c_\alpha}{v'} + a_\alpha \right) / tz$.

Consider a marginal change in the enforcement parameter, α. The left-hand side of (29) measures the marginal revenue valued in terms of the benefit from the resulting expenditure. The right-hand side measures the marginal cost of raising that revenue: this includes the marginal compliance cost borne by the individual and the marginal administrative cost, where the latter is again valued in terms of marginal opportunity cost (of spending the revenue on administration rather than on the public expenditure). Thus, an equi-marginal condition again applies: for a given tax rate, the marginal benefits and costs of raising α must be equal at the optimum. The revenue component, $\frac{t\partial z}{\partial \alpha}$, and the marginal valuation, v', are likely to fall as α increases, while the marginal costs are likely to increase. Hence α is increased until the two sides of (29) are equal.

Importantly, in the KS specification, the change in taxable income resulting from a change in α arises only from a change in the amount of income concealed from the tax authorities, e. This occurs because an increase in α causes the cost of concealement, $c(e, \alpha)$, to rise, so that e falls and z increases correspondingly. There is no direct effect on labour supply of a change in α because the disutility of work is a function only of h. This feature is considered further in the following section.

3 Tax Administration and Labour Supply

It was mentioned above that the KS model does not include the possibility, stressed by Gemmell and Hasseldine (2014), that tax administration may have a direct effect on labour supply. This can be introduced in the KS model by the simple expedient of making disutility, ϕ, a function of hours worked and enforcement, so that $\phi = \phi(h, \alpha)$.[12] By increasing the disutility of work, increased enforcement by the authorities is likely to provide an incentive to work less.[13]

In examining the first of the first-order conditions above, giving the optimal tax

[12] A direct effect would arise if, for example, the utility function were written as multiplicative (that is, in Cobb-Douglas form), rather than additive. However, this would considerably complicate the analysis.

[13] Gemmell and Hasseldine (2014) suggest that increased enforcement has the effect of raising the effective marginal tax rate, thereby producing the types of response included in the conventional elasticity of taxable income.

rate conditional on α, it is clear that this is not affected by the modification in ϕ. The government's social welfare function is:

$$W = \{wh - t\,(wh - e) - c\,(e, \alpha)\} - \phi\,(h, \alpha) + v\,(s) \tag{30}$$

Totally differentiating with respect to α and s gives:

$$dW = \left\{ w\,(1-t)\,\frac{\partial h}{\partial \alpha} - c_\alpha - \frac{\partial \phi\,(h, \alpha)}{\partial \alpha} \right\} d\alpha + v' ds \tag{31}$$

Writing:

$$\frac{\partial \phi\,(h, \alpha)}{\partial \alpha} = \frac{\partial \phi\,(h, \alpha)}{\partial h}\frac{\partial h}{\partial \alpha} \tag{32}$$

and setting $dW = 0$ gives the marginal rate of substitution along the social indifference curve as:

$$\left.\frac{ds}{d\alpha}\right|_W = \left(\frac{1}{v'}\right)\left[c_\alpha + \frac{\partial h}{\partial \alpha}\left\{\frac{\partial \phi\,(h, \alpha)}{\partial h} - w\,(1-t)\right\}\right] \tag{33}$$

However, from (6), the term in curly brackets in (33) is zero, so that:

$$\left.\frac{ds}{d\alpha}\right|_W = \frac{c_\alpha}{v'} \tag{34}$$

This is the same as (24), so that, in combination with the slope of $ds/d\alpha = tz_\alpha - a_\alpha$ along the government's budget constraint, as in (25), the first-order condition for optimal α looks exactly the same as (28) or its more easily interpreted form in (29). The crucial difference is that the term, z_α, is now:

$$z_\alpha = w\frac{\partial h}{\partial \alpha} - e_\alpha \tag{35}$$

rather than simply $-e_\alpha$. It depends on the change in labour income as well as the change in the amount concealed. As discussed above, the term, e_α, is negative, so that its effect is to raise z_α. However, if $\frac{\partial h}{\partial \alpha} < 0$, as suggested, then the effect on taxable income is reduced: indeed it is possible in principle for the direct negative labour supply effect to outweigh the positive enforcement effect on taxable income.

The disutility-enhancing effect of tax enforcement, thereby introducing a direct labour supply effect, $\frac{\partial h}{\partial \alpha}$, may therefore appear at first sight to have no effect on the first-order condition for optimal α, expressed in terms of z_α. But where z_α now includes the term $w\frac{\partial h}{\partial \alpha}$ as well as (the negative of) e_α, the optimal value of α is likely to be smaller than in the basic KS model.

4 The Compliance Gap

An optimum degree of tax enforcement clearly carries with it the concept of an optimum compliance gap, g, defined as the proportional difference between the tax that should legally be paid and the tax actually paid. Hence:

$$g = \frac{twh - tz}{twh} \tag{36}$$

and since $z = wh - e$, this becomes:

$$g = \frac{e}{wh} \tag{37}$$

In the KS model, it has already been stressed that z_α depends only on the change in e, so that $z_\alpha = -e_\alpha$. Converting this to elasticities gives the result, in KS's equation (12), that:

$$\eta_{z,\alpha} = -\left(\frac{e}{z}\right)\eta_{e,\alpha} \tag{38}$$

Hence:

$$\frac{e}{z} = -\frac{\eta_{z,\alpha}}{\eta_{e,\alpha}} \tag{39}$$

Using (37), it can be seen that:

$$\frac{g}{1-g} = \frac{e}{z} \tag{40}$$

so that, using (39):

$$\frac{g}{1-g} = -\frac{\eta_{z,\alpha}}{\eta_{e,\alpha}} \tag{41}$$

Which is the result in KS, equation (13). Equation (28) above gives the optimal $\eta_{z,\alpha}$ in terms of marginal compliance and administrative costs, among other things. Hence (41) can be converted into a statement about the optimal compliance gap.

This KS result can be extended to cover the case discussed in Section 3 where tax enforcement also affects labour supply. In this case it is necessary to convert (35) into elasticity form, so that:

$$\eta_{z,\alpha} = \frac{wh}{z}\eta_{h,\alpha} - \frac{e}{z}\eta_{e,\alpha} \tag{42}$$

and using $wh/z = 1 + \frac{e}{z}$, it can be seen that:

$$\eta_{z,\alpha} = \eta_{h,\alpha} + \frac{e}{z}\left(\eta_{h,\alpha} - \eta_{e,\alpha}\right) \tag{43}$$

and:

$$\frac{e}{z} = \frac{\eta_{z,\alpha} - \eta_{h,\alpha}}{\eta_{h,\alpha} - \eta_{e,\alpha}} \tag{44}$$

which can be substituted into (40). It is useful to solve for the optimal gap, rather than $g/(1-g)$, as follows:

$$g = \frac{\eta_{z,\alpha} - \eta_{h,\alpha}}{1 + \eta_{h,\alpha} - \eta_{e,\alpha}} \tag{45}$$

remembering that $\eta_{z,\alpha}$ is taken from the right-hand side of (28) and all elasticities are evaluated at the optimum tax rate. As $\eta_{h,\alpha} < 0$, it is clear that the optimum gap is larger than when there are no direct labour supply responses to α.

5 Many Individuals

The KS model, by focussing on a single individual, obviously has no redistributive role for taxation. Furthermore, government expenditure, s, involves a 'private good' which provides a higher marginal utility for the individual than consumption, x, financed from post-tax-and-compliance income. The individual's resistance to paying tax comes only from the distortion to the choice between leisure and private consumption, given that, by assumption, the publicly financed good has no effect on labour supply. The government's 'social welfare function' coincides precisely with the utility function of the individual.

The present section shows how the seemingly highly restrictive assumption of a single individual can be relaxed. The model is extended to allow for many individuals, each facing a different wage rate and having different preferences, although the quasi-linear nature of the utility function is retained. The government's welfare function is also modified to allow for distributional judgements, and a transfer payment in the form of a basic income is included so that the tax operates as a redistributive tax and transfer system. In addition, public expenditure is modelled as a pure public good.

The aim is to consider the conditions under which the basic KS results carry over to appropriately specified aggregates.

First, it is clear that an exogenously determined universal or 'basic' income can easily be introduced. This provides a redistributive role: the income tax then becomes a 'basic income – flat tax' structure. But it has no effect on the first-order conditions for determining t and α, though of course it affects the actual solutions via the government's budget constraint (and the tax rate is constrained to be above the value needed to finance the basic income). The quasi-linear utility functions ensure that it has no effect on individuals' labour supplies: that is, labour supply is not subject to income effects. Hence in what follows, the basic income is not included, for simplicity only.

Suppose there are N individuals, and individual variables are given a subscript i, for $i = 1, ..., N$. Consider first the tax rate, for a given enforcement parameter, α. The government budget constraint becomes:

$$\begin{aligned} s &= t\sum_{i=1}^{N}(w_i h_i - e_i) - a(\alpha) \\ &= t\sum_{i=1}^{N} z_i - a(\alpha) \end{aligned} \tag{46}$$

where the administrative cost, $a(\alpha)$, is a measure of the aggregate cost. Along this constraint:

$$\frac{ds}{dt} = \sum_{i=1}^{N}(z_i + t z_{i,t}) \tag{47}$$

where $z_{i,t} = \partial z_i / \partial t$.

The welfare function that the government seeks to maximise, W, must be modified to deal with the many individuals. Suppose first that public expenditure is on a 'pure' public good: it is consumed without rivalry in equal amounts by each individual, although individuals need not have the same preferences. The government takes a view about the 'social value', denoted $V(s)$, of this expenditure. Next, suppose the government is concerned with the private expenditure, x_i, of individuals, and attaches a 'welfare weight' of ξ_i to each value, with $\sum_{i=1}^{N}\xi_i = N$. The welfare function then takes the form:

$$W = \sum_{i=1}^{N} \xi_i x_i + V(s) \tag{48}$$

Totally differentiating (48) with respect to t and s, and noting that $\partial x_i / \partial t = z_i$:

$$\left.\frac{ds}{dt}\right|_W = \frac{\sum_{i=1}^{N} \xi_i z_i}{V'} \tag{49}$$

where $V' = \partial V(s) / \partial s$. Equating (47) and (49):

$$\frac{\sum_{i=1}^{N} \xi_i z_i}{V'} = \sum_{i=1}^{N} (z_i + t z_{i,t}) \tag{50}$$

Define the aggregates, $Z = \sum_{i=1}^{N} z_i$, $Z_t = \sum_{i=1}^{N} z_{i,t} = \partial Z / \partial t$ and $\widetilde{Z} = \sum_{i=1}^{N} \xi_i z_i$. The latter is a welfare-weighted aggregate taxable income: these terms could of course easily be converted into means rather than aggregates, if desired. Hence the first-order condition is:

$$\frac{\widetilde{Z}}{V'} = Z + t Z_t \tag{51}$$

The result in (51) can be compared with (14). Diving the the right-hand side by Z gives the aggregate elasticity of tax revenue, R_T, with respect to the tax rate, t, which can be denoted, $\eta_{R_T,t}$. Hence, at the optimum:

$$\frac{\widetilde{Z}}{Z} = V' \eta_{R_T,t} \tag{52}$$

In the situation where the government does not care about inequality, all $\xi_i = 1$ and the left-hand side of (52) is 1, which clearly corresponds to the result in the basic KS model. In the case where higher weights are attached to relatively lower taxable incomes, the ratio $\frac{\widetilde{Z}}{Z}$ is less than one and this produces a relatively higher optimal tax rate (for given α). Indeed, it is possible to define an inequality measure, I_Z, as $I_Z = 1 - \frac{\widetilde{Z}}{Z}$, following the approach of Gini and Atkinson measures, where $\widetilde{Z}$ can be regarded as a type of equally-distributed equivalent measure of total taxable income. Then $\frac{\widetilde{Z}}{Z} = 1 - I_Z$ is a measure of equality. With either homogeneous individauls (corresponding to the single-individual case) or indifference on the part of government to the distribution, the condition in (52) reduces to the KS case.

Turning to the first-order condition for α, total differentiation of W with respect to α and s gives the marginal rate of substitution along the government's social indifference curves as:

$$\left.\frac{ds}{d\alpha}\right|_W = \frac{\sum_{i=1}^N \xi_i c_{i,\alpha}}{V'} \tag{53}$$

where now $c_{i,\alpha} = \partial c_i / \partial \alpha$. Along the government's budget constraint:

$$\frac{ds}{d\alpha} = t \sum_{i=1}^N z_{i,\alpha} - a_\alpha \tag{54}$$

Equating (53) and (54) gives:

$$\begin{aligned} V'\left(tZ_\alpha - a_\alpha\right) &= \sum_{i=1}^N \xi_i c_{i,\alpha} \\ &= \widetilde{C}_\alpha \end{aligned} \tag{55}$$

The term, $\widetilde{C}_\alpha$, is a welfare-weighted change in total compliance costs. A slight rearrangement of this equation gives the result that:

$$V' t \frac{\partial Z}{\partial \alpha} = \widetilde{C}_\alpha + V' a_\alpha \tag{56}$$

This optimality condition clearly has the same kind of interpretation as the equimarginal condition in (29), except that the terms refer to aggregates and the marginal compliance cost allows for distributional considerations. Clearly, when all $\xi_i = 1$, for $i = 1, ..., N$, $\widetilde{C}_\alpha = C_\alpha$. If more weight is attached to lower values of $c_{i,\alpha}$, $\widetilde{C}_\alpha < C_\alpha$ and the optimum value of α is likely to be lower than when the government is indifferent to the distribution.

6 Explicit Functional Forms

It is instructive to introduce explicit functional forms, which allows closed-form solutions to be obtained and examined. This section illustrates the properties of the model using the simplest forms which are tractable.[14] The basic single-person KS model is illustrated, where disutility from work is a function of hours worked only, rather than the augmented model of Section 3.

[14] Linear forms are tractable but for obvious reasons are of no interest.

6.1 The Individual's Choice

First, consider the quadratic form:[15]

$$\phi(h) = \beta h^2 \tag{57}$$

whereby the disutility from work increases with the square of the time spent working. From the condition (6):

$$2\beta h = w(1-t) \tag{58}$$

and the optimal labour supply for the individual is given by:

$$h = \frac{w(1-t)}{2\beta} \tag{59}$$

Thus hours worked are a simply linear (declining) function of the proportional tax rate. This is illustrated in Figure 3, showing labour supply falling at a constant rate from $w/2\beta$ to 0 as t increases from 0 to 1. Also, labour supply is not affected in this case by the value of α.

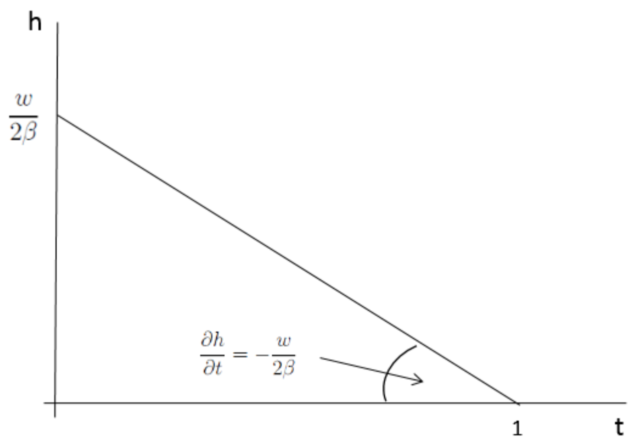

Figure 3: Schedule of h Plotted Against t

Second, suppose the cost of concealing an amount, e, of taxable income is given by:

$$c(e,\alpha) = \alpha e^2 \tag{60}$$

[15] Quadratic utility functions have a long tradition in economic models, dating from Launhardt's 1885 explicit derivation of demand and supply functions in a two-good, two-person version of Walras's model.

Using (8), the optimal value of e, for given α, is:

$$e = \frac{t}{2\alpha} \tag{61}$$

Hence the amount of income concealed is directly proportional to the tax rate and inversely proportional to the enforcement parameter, α.

Taxable income, $z = wh - e$, is thus:

$$z = \frac{w^2(1-t)}{2\beta} - \frac{t}{2\alpha} \tag{62}$$

Hence:

$$\frac{\partial z}{\partial t} = -\frac{1}{2}\left(\frac{w^2}{\beta} + \frac{1}{\alpha}\right) \tag{63}$$

The relationship between z and t is illustrated in Figure 4. In this case a higher value of α causes the linear relationship to pivot upwards about the point $w^2/2\beta$, its value when $t = 0$: this is shown by the dashed line.

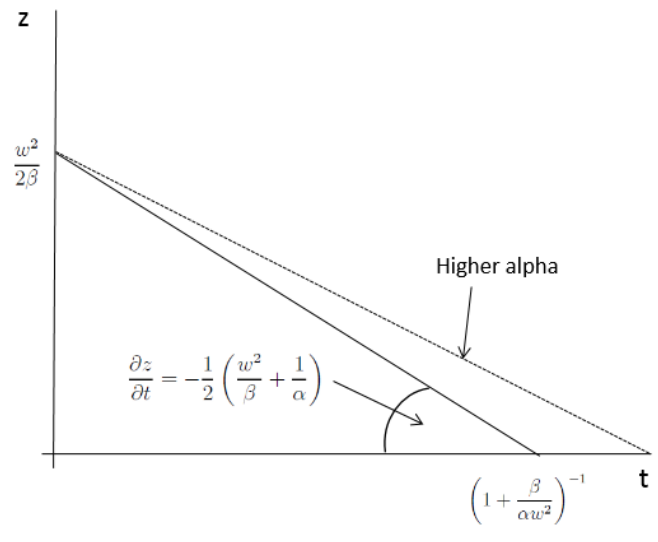

Figure 4: Schedule of z Plotted Against t

The elasticity $\eta_{z,t}$ thus falls from zero when $t = 0$ to $-\infty$ when $t = \frac{1}{2}\left(1 + \frac{\beta}{\alpha w^2}\right)^{-1}$. Hence, $\eta_{z,1-t} = -\left(\frac{1-t}{t}\right)\eta_{z,t}$ is undefined when $t = 0$, and becomes positive and large as

t approaches the value (less than 1) where $z = 0$. Using the above results, the elasticity of taxable income is therefore:

$$\eta_{z,1-t} = \frac{(1-t)\left(\frac{w^2}{\beta} + \frac{1}{\alpha}\right)}{\frac{w^2}{\beta} - t\left(\frac{w^2}{\beta} + \frac{1}{\alpha}\right)} \tag{64}$$

which can be rewritten as:

$$\eta_{z,1-t} = \frac{(1-t)\frac{w^2}{\beta} + \frac{(1-t)}{\alpha}}{(1-t)\frac{w^2}{\beta} - \frac{t}{\alpha}} \tag{65}$$

Hence in this model $\eta_{z,1-t} > 1$. And with such a large elasticity of taxable income, the excess burden of the tax is high.

Consider also the relationship between x and h. From 4, the individual's budget constraint is a straight line, shown in Figure 5, with a slope of $w(1-t)$. The value of x becomes zero for $h = (c(e, \alpha) - et)/w(1-t)$, and so the budget constraint shifts downwards as α increases. The individuals tangency position is shown in the diagram, and as h does not depend on α, a higher value of α is associated with a vertical downward shift in x: this is shown by the dashed indifference curve and budget line in Figure 5.

6.2 Optimal Policy

Substituting for $\eta_{z,1-t}$ from (65) into the optimality condition in (17), gives:

$$\frac{v' - 1}{v'} = \frac{t\left(\frac{w^2}{\beta} + \frac{1}{\alpha}\right)}{\frac{w^2}{\beta} - t\left(\frac{w^2}{\beta} + \frac{1}{\alpha}\right)} \tag{66}$$

Further rearrangement gives the optimal tax rate as:

$$t = \frac{w^2}{\left(w^2 + \frac{\beta}{\alpha}\right)\left(1 + \frac{v'}{v'-1}\right)} \tag{67}$$

In the case where $v' = 1$, the public expenditure gives no advantage over private expenditure and hence the optimal tax rate is zero. This expression gives the optimal tax rate in terms of the exogenous wage rate and the parameter β, along with the value

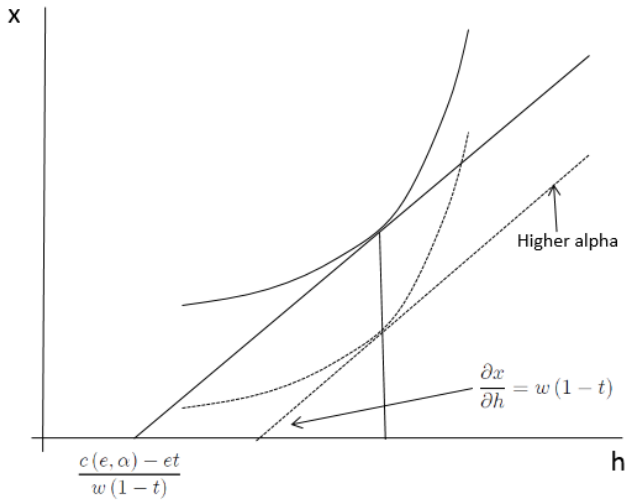

Figure 5: Schedule of x Plotted Against h

of the other policy parameter α, and the value of v'. This is considerably simplified if it is assumed that the marginal utility of public expenditure is constant, such that:

$$v(s) = ks \tag{68}$$

and $v' = k$. Without this simplification, it would be necessary to use the government's budget constraint to obtain the value of s, and the algebra becomes much more messy. Furthermore, as k increases, and as β/α is small relative to w^2, t approaches 0.5. Hence, even where the public expenditure has a high marginal valuation, the optimal rate does not exceed 0.5,

The optimal choice in the general case was shown in Figure 2. In the present case the schedules of s against t take the form shown in Figure 6. Here the government's budget constraint, $s = tz - a$, becomes:

$$s = t\left[\frac{w^2}{2\beta} - \frac{t}{2}\left(\frac{w^2}{\beta} + \frac{1}{\alpha}\right)\right] - \delta\alpha^2 \tag{69}$$

and this can be shown to read a maximum when t takes the value given by:

$$t = \frac{1}{2}\left(1 + \frac{\beta}{\alpha w^2}\right)^{-1} \tag{70}$$

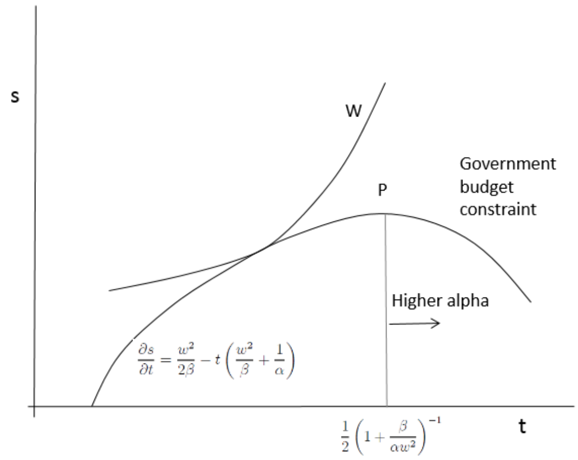

Figure 6: Schedule of s Plotted Against t

As this does not depend on α, the peak, at P, simply shifts to the right as α increases.

From (67) it can also be seen that as α increases, t (asymptotically) approaches $(k-1)/(2k-1)$. However, (67) is just one equation, from the condition in (17). It is also necessary to consider the condition in (28). This requires specification of the functions, $c(e, \alpha)$ and $a(\alpha)$. Suppose again that this follows the quadratic:

$$c(e, \alpha) = \alpha e^2 \tag{71}$$

so that, using (61):

$$\begin{aligned} c_\alpha &= e^2 \\ &= \left(\frac{t}{2\alpha}\right)^2 \end{aligned} \tag{72}$$

Suppose the enforcement cost is quadratic in α, so that:

$$a(\alpha) = \delta\alpha^2 \tag{73}$$

and $a_\alpha = 2\delta\alpha$.

The relevant relationships between s and α are illustrated in Figure 7. Using (24):

$$\left.\frac{ds}{d\alpha}\right|_W = \frac{c_\alpha}{v'}$$
$$= \frac{1}{k}\left(\frac{t}{2\alpha}\right)^2 \tag{74}$$

Hence, social indifference curves are upward sloping but concave (since $\frac{d^2s}{d\alpha^2}$ is negative). Along the budget line, $\frac{ds}{d\alpha} = t\frac{\partial z}{\partial \alpha} - \frac{\partial a}{\partial \alpha}$, and this has a turning point (maximum) when $\alpha = \left(\frac{t^2}{4\delta}\right)^{\frac{1}{3}}$.

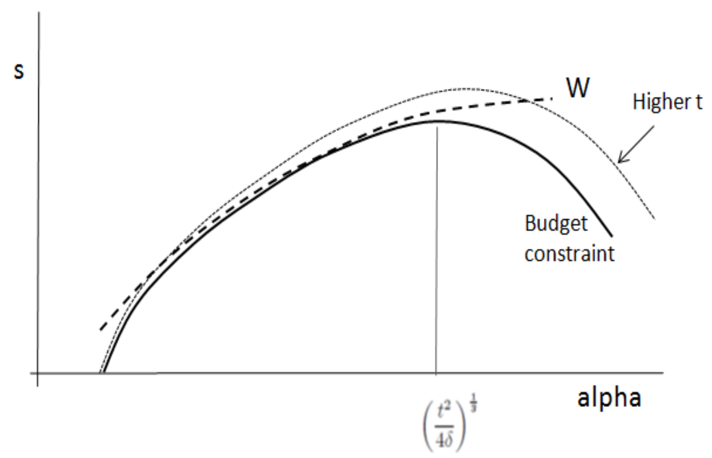

Figure 7: Schedule of s Plotted Against α

Next, obtain $\eta_{z,\alpha}$ by differentiation of (62) to get:

$$\eta_{z,\alpha} = \frac{t}{2\alpha z} \tag{75}$$

and substituting for z:

$$\eta_{z,\alpha} = \left[w^2\frac{\alpha}{\beta}\left(\frac{1-t}{t}\right) - 1\right]^{-1} \tag{76}$$

Substituting into (28) gives:

$$\frac{t}{2\alpha z} = \alpha\left(\frac{\frac{1}{k}\left(\frac{t}{2\alpha}\right)^2 + 2\delta\alpha}{tz}\right) \tag{77}$$

Table 2: Parameter Values

Parameter	Value
δ	20
$k = v'$	1.3
β	0.7
w	65

After rearrangement, this becomes:

$$\alpha^3 = \frac{t^2\left(2 - \frac{1}{k}\right)}{8\delta} \tag{78}$$

Which expresses α in terms of t and the exogenous variables k and δ. Finally the solutions for the government's policy variables, t and α can be obtained by solving the two simultaneous equations (78) and (67).

Even for this 'simple' model, these two nonlinear equations need to be solved numerically. To make the properties more transparent, it is instructive to consider the two equations separately. Suppose the parameters take the values shown in Table 2: as shown below these give sensible values of the various endogenous variables.

The two profiles are shown in Figure 8. Series 1 and 2 are respectively (67) and (78). The former displays the asymptotic tendency mentioned above, while the second is approximately linear. These two profiles intersect at $t = 0.187$ and $\alpha = 0.066$.

These values imply, at the optimal policy, $h = 37.75$, $z = 2452.01$, and $e = 1.42$, incurring a cost of $c = 0.13$. The (endogenous) elasticity of taxable income is $\eta_{z,1-t} = 1.00$. This high elasticity gives rise to a welfare cost per dollar of revenue of 1.23. The elasticity, $\eta_{z,\alpha} = 0.0006$. The optimal value of $a(\alpha)$ is 0.087. Gross tax revenue is 458.60.

6.3 A 'Four Quadrant' Diagram

Another way to view the model's structure is to combine several of the schedules shown above into the (quasi) four-quadrant diagram of Figure 9. This reveals the nature of the optimal solutions for α and t, but of course illustrating the optimum positions in this way is far from providing a simple comparative-static tool. This is because the

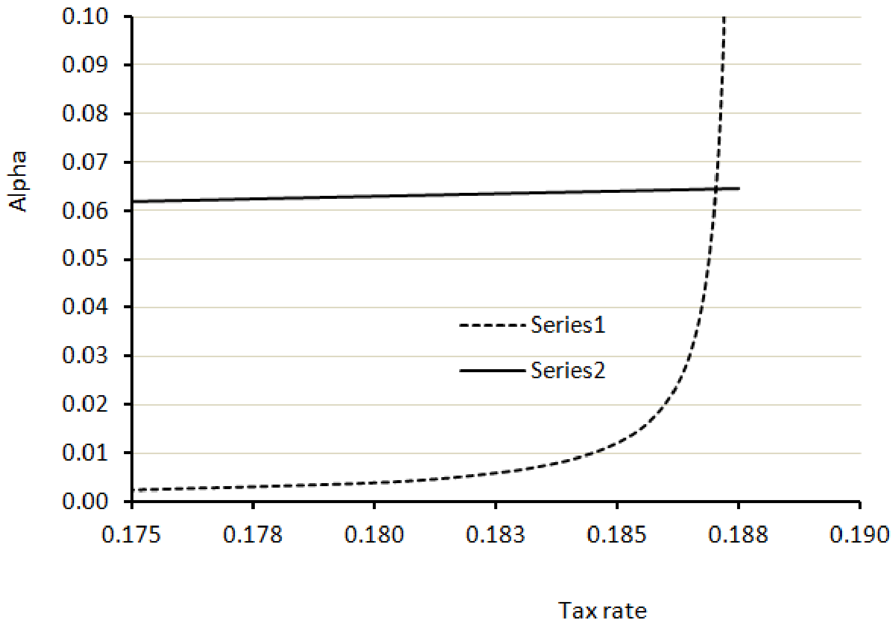

Figure 8: Optimal α and t

various schedules are conditional on values of variables in other quadrants (except that labour supply in the south-east quadrant is not affected by α). Hence, for example, a movement along the government's budget constraint in the north-east quadrant (as t changes) involves a shift in the constraint shown in the north-west quadrant. The budget line in the south-west quadrant shifts as α changes, and its slope changes as t changes.

7 Conclusion

This paper has provided a more elementary derivation of the main results in the Keen and Slemrod (2016) analysis of optimal tax administration. It also considers explicit functional forms and demonstrates how solutions can be obtained for the optimal tax rate and enforcement parameter: the solution to two simultaneous nonlinear equations. It is hoped that this is useful for those who may be interested in applying or extending the basic model.

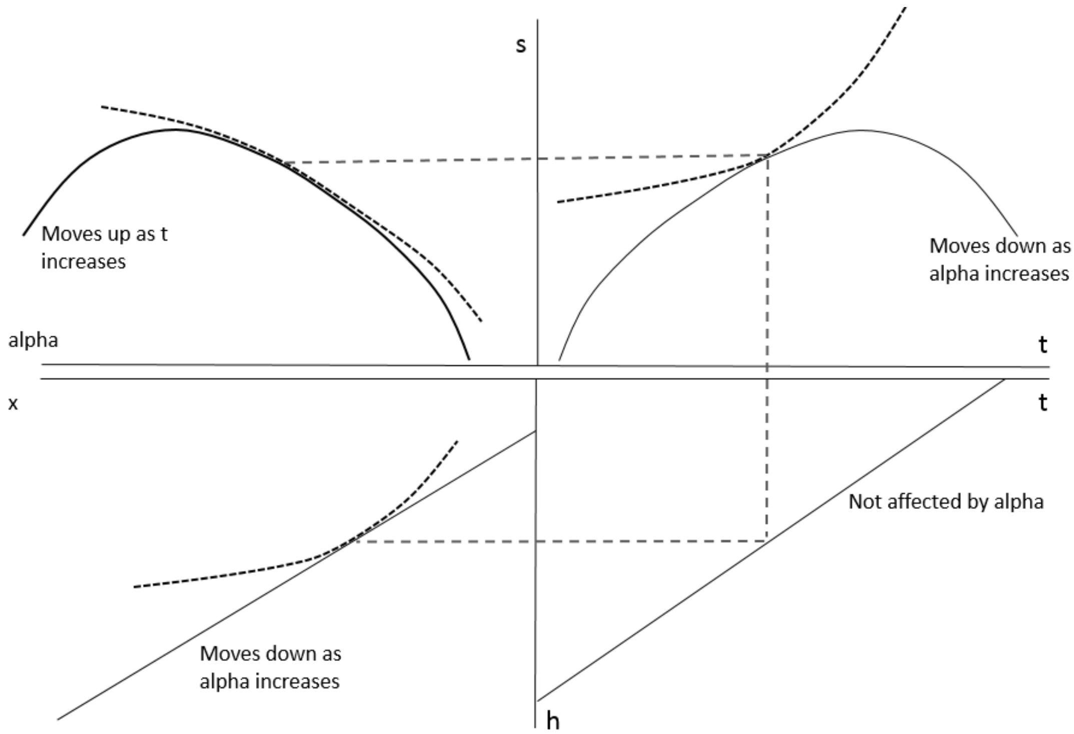

Figure 9: A 'Four Quadrant' Representation of an Optimal Policy

Several extensions were provided. For each of the first-order conditions, relating to the optimal tax rate for given enforcement parameter, and optimal enforcement parameter for given tax rate, intuitive explanations were provided in terms of equi-marginal conditions relating marginal costs and benefits of changing the relevant variable. The model was extended in the simplest way to allow for a direct effect on labour supply of an increase in tax enforcement. This was also found to have implications for the optimal compliance gap. Finally, the single-person model of Keen and Slemrod was extended to cover a many-person case. The government's objective was modified to allow for a concern for distributional outcomes, and a basic income was added to provide a progressive tax-transfer scheme. Modified first-order optimality conditions were derived in terms of relevant aggregates and welfare-weighted aggregates.

The analysis raises a query concerning the two elasticities that play such a central role in Keen and Slemrod's discussion of the two simultaneous first-order conditions:

these are the widely used elasticity of taxable income and their new concept of the elasticity of taxable income with respect to the enforcement parameter. In common with some other optimal tax models, elegant first-order conditions can be obtained giving some useful insights into the nature of an optimal solution, and here these conditions depend on the two elasticities. But a crucial feature of the model is that the elasticities are themselves endogenous. Furthermore, more intuitive results are obtained in terms of marginal costs and benefits of tax rate and tax enforcement changes. Hence, it perhaps calls into question whether the elasticities should be the main focus of analysis, rather than the more fundamental relationships in the model.[16]

[16] Marshall, who was largely responsible for introducing the concept of elasticity into economics, also warned (in the context of trade) against an assumption of constancy; see Marshall (1923, p. 388). A further example is provided by the analysis of labour supply. The KS analysis eschews using a concept of the net-wage labour supply elasticity, which is not constant in the model. In models with nonlinear tax functions, the elasticity can vary enormously for a single individual. Empirical studies no longer focus on estimating 'the elasticity', but instead use structural discrete choice models. Yet, many discussions find it useful to continue to use the concept.

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